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Laminar free convection heat transfer to fluids from an isothermal sphere.

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LAMINAR FREE CONVECTION HEAT TRANSFER TO FLUIDS FROM AN ISOTHERMAL SPHERE

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ABSTRACT

An experimental investigation of laminar free convection heat transfer to water and non-Newtonian fluids from an isothermal sphere has been carried out. The experimental data obtained from water were used to verify the theoretical equations of Merk and Prins (40) and Acrivos (1). Thirteen non-Newtonian aqueous polymer solutions were employed to investigate the effect of non-Newtonian behaviour on free convection heat transfer. The thirteen solutions were 0.5%, 1.0% and 1.5% Carbopol 934, 0.25%, 0.5% and 1.0% Carbose IM, 0.5%, 1.0%, 1.5% and 2.5% Carbose IM with 0.1% Sodium Benzoate and 0.25%, 0.75% and 1.25% Natrosol 250H with 0.1% Sodium Benzoate. It was found that very viscous solutions such as 1.5% Carbopol 934 and 2.5% Carbose were not suitable for free convection heat transfer study due to the formation of a film on the heat transfer medium.

The variation of physical properties with temperature was found to have a negligible effect on the theoretical equations. The heat transfer data of water were correlated in the form:

$$\overline{N_{Nu}} = C' (N_{Gr} N_{Pr})^{1/4}$$

The C' value was found to be 0.494 using the radius of the sphere as the characteristic length in the correlation. This value is in good agreement with the predictions of Merk and Prins (49) and Acrivos (1) at the following ranges of Nusselt, Grashof and Prandtl numbers.

$$14.7 \leq N_{Nu} \leq 57$$

$$8.8 \times 10^5 \leq N_{Gr}N_{Pr} \leq 3.4 \times 10^8$$

For the non-Newtonian polymer solutions, Acrivos equation was used to correlate the data. The results are as follows:

$$\overline{N_{Nu}} = 0.561\ N_{Gr}^{\frac{1}{2(n+1)}}\ N_{Pr}^{\frac{n}{3n+1}}$$

$$2.5 \leq \overline{N_{Nu}} \leq 55.5$$

$$7.99 \times 10^2 \leq N_{Gr}N_{Pr} \leq 1.33 \times 10^8$$

$$0.66 \leq n \leq 0.99$$

$$10 \leq K \leq 24163$$

The heat transfer data were also correlated by an empirical equation which has the form of

$$\overline{N_{Nu}} = C_1 (N_{Gr}N_{Pr}')^{C_2}$$

The values of C_1 and C_2 were found to be 0.611 and 0.241 respectively at the following range of $N_{Gr}N_{Pr}'$.

$$6.0 \times 10^2 \leq N_{Gr}N_{Pr}' \leq 1.6 \times 10^8$$

Comparison of the results of correlations shows that Acrivos' equation is slightly better than the empirical equation.

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I. INTRODUCTION

Most chemical and physical processes in industries involve the transport of momentum, energy and mass. One of the most commonly encountered phenomena is free convection. Free convection is caused by the difference in fluid density due to the temperature gradient between the heating or cooling surface and the fluid. It is therefore reasonable to expect the temperature difference , thermal expansion coefficient and density of the fluid to be important factors in the phenomenon of free convection. As early as the nineteenth century, the problem of free convection in Newtonian fluids was studied and the problem is now well explored, at least for heat surfaces with simple geometries.

In recent years, the increasing importance of solutions and melts of high polymers, suspension of solids in liquids, emulsions etc., as raw materials or products in a large variety of industrial processes, has stimulated a considerable amount of interest in the behaviour of non-Newtonian fluids when in motion. In particular the mechanisms of momentum and energy transfer in non-Newtonian fluids has been studied intensely. A summary of the results of these studies are avalaible (50,80).

Of particular interest in the present study is the problem of free convection heat transfer to water and non-Newtonian fluids from an isothermal sphere. The aims of this study are threefold:

(i) To measure the free convection heat transfer rates from an isothermal sphere to water and non-Newtonian fluids.

(ii) To test the theoretical equations proposed by Merk and Prins (49) for Newtonian fluids, by Acrivos (1) for both Newtonian and non-Newtonian fluids and the empirical correlation (61,78) for non-Newtonian fluids with a form similar to that established for Newtonian fluids.

(iii) To investigate the degree of non-Newtonian fluid behaviour on the phenomenon of free convection.

II. LITERATURE SURVEY

Literature surveys have been carried out in three areas relevant to this study:

i. Free convection from a sphere to Newtonian fluids

ii. Free convection to non-Newtonian fluids

iii. Thermal conductivity of liquids

A. Free Convection from A Sphere to Newtonian Fluids

One of the very early theories proposed by Langmur (46) and Rice (66) to explain free convection from heated surfaces is the film theory. In free convection, it is well known that the liquid film immediately in contact with the heating surface has zero velocity and heat must be transmitted through this layer by conduction. The film theory extended this idea by defining an equivalent heat conduction coefficient so that convection from any surface may be expressed in terms of the equivalent conduction coefficient through an imaginary stagnant air film of finite thickness across which the entire temperature drop occurs. Experimental work has been carried out from a sphere and a cylinder to air by Eleenbaas (21) to prove this theory. The results were later extended to other solutions by the same investigator (22).

There are two major arguments against the use of this theory:

(1). The calculation is rendered more cumbersome because it is first necessary to calculate the film thickness to get the heat flow through the conduction equation.

(2). It does not properly describe the actual mechanism of the process.

Because of these disadvantages, the use of this theory was gradually abandoned.

The general equations for the problem of free convection have also been studied by Nusselt (52,53), Davis (18), Fishenden and Saunders (27) and others. The results are usually reducible to the form of

$$\overline{N_{Nu}} = f_1(N_{Gr})\ f_2(N_{Pr}) \tag{2.1}$$

Although the functions of f_1 and f_2 are not necessarily the same, it has been shown both theoretically and by correlations of the experimental data that, as a reasonable approximation, eqn. (2.1) can be simplified to eqn. (2.2).

$$\overline{N_{Nu}} = f_3(N_{Gr}N_{Pr}) \tag{2.2}$$

The correlation of Jakob and Linke (39) in 1933 for free convection from various surfaces including a sphere, is in the form of eqn. (2.2). Theoretical analysis on the heat loss through laminar free convection was made by Merk and Prins (49) in 1954 for a sphere, cylinder and flat plate by the approximation method. Predictions which are given for large Prandtl numbers by the first and second approximation method, will be used later to compare with the experimental results for water. Acrivos (1) in 1960 solved the boundary layer equations for free convection from a sphere and flat plate by the asymptotic methods. His prediction will also be tested.

The free convection of air from a sphere has been studied by Yuge (28). Kyte, Madden and Piret (44) made a very interesting study on free convection from both sphere and cylinder at reduced pressure. It was found that at low gas pressure, the thickness of the convective boundary layer is large and the effect of free convective conduction is important.

Ranz and Marshall (10) in 1952 investigated the factors influencing the evaporation of liquid drops in the laminar region. Vanier (86) and Schank and Schenkels (74) have studied thermal free convection by melting an ice sphere in water. Van der-Burgh (85) in 1960 and Kranse and Schenk (40) in 1965 studied free convection by dissolving a sphere of solid benzene in a large quantity of liquid benzene with a uniform temperature. Schenkels and Schenk (75) also studied the same problem by dissolving organic spheres. This method of studying free convection by melting solid materials has proven to be useful where a uniform and well defined surface temperature is desired.

Boberg and Starrette(9) determined the free convection heat transfer properties of fluids by the transient response method using a copper sphere in water and claimed this method can be used commercially to determine free convection properties of new fluids. Agrawal and Adelman (2) studied free convection from an isothermal copper sphere heated by fluid while Amato and Tien (5) investigated the same problem with an electrically heated sphere.

Theoretical studies of laminar free convection in boundary layers for axisymmetric bodies are also given elsewhere (72,73).

B. Laminar Free Convection to non-Newtonian Fluids

One of the very first solutions of laminar free convection in non-Newtonian fluids which employed boundary layer theory was presented in 1960 by Acrivos (1) to predict the local as well as the average free convection from surfaces with simple geometry. This equation will be tested with the present experimental results.

St. Pierre and Tien (81) carried out experimental work on free convection to non-Newtonian fluids in a confined space. Reilly and Adelman (61) used different sizes of vertical and horizontal plates to investigate the effect of non-Newtonian behaviour in free convection; Sharma and

Adelman (78) extended the work to solutions of higher concentration. Approximation solution for laminar free convection between a vertical plate and a power-law fluid with high Prandtl number was obtained by Tien (82) and available data (61) agreed well with this solution. Tien and Tsuei (89) used the integral method to find the approximation solution for heat transfer from a vertical plate to an Ellis' fluid which was also compared with the data from Reference (61). Agrawal (2) and Amato (8) performed free convection experiments using a copper sphere. Buoyancy-driven convection in horizontal layers of polymer solution in a confined space was also studied by Liang and Acrivos (47) to find the heat transfer curve and flow pattern of the fluid.

C. Thermal Conductivity of Liquids

The available experimental methods for determining the thermal conductivity of liquids are reviewed in References (13) and (14) to 1953. Further review up to 1963 is found in Reference (38). It was found that the method of Riedel (65), which is employed in the present study, is still very desirable compared to other methods such as (16,37,38).

III. THEORY

A. Classification of Fluid Behaviour

The classical theory of fluid dynamics was developed from studies of imaginary ideal fluid that was incompressible and without viscosity or elasticity. Detailed mathematical relationships have been obtained for the behaviour of an ideal fluid in a variety of physical situations (45). Some of these results have proven to be useful approximations to the performance of real fluids in certain cases, but their practical applications were substantially limited until the introduction of the boundary layer concept of Prandtl.

In essence, Prandtl showed that the frictional effects are confined to a relatively thin fluid layer adjacent to the solid surfaces, this layer being the boundary layer. The flow outside the boundary layer may be regarded as frictionless, so that the relationship developed for an ideal fluid will prevail in this region.

Some limitations of the application of ideal fluid theory led to the development of a dynamic theory for the simplest class of real fluids — those which are commonly described as Newtonian.

In recent years, the increasing importance in process industries and elsewhere of material whose flow behaviour in shear cannot be described by Newtonian relationships invoked the development of another stage of dynamic theory. Examples of non-Newtonian fluids include solutions and melts of high polymers, suspensions of solids in liquids, emulsions and materials possessing both viscous and elastic properties.

Industries in which non-Newtonian behaviour will be encountered include petroleum, soap and detergents, pharmaceutical, atomic energy, paper and pulp, paint, light and heavy chemicals, ore processing etc. It is therefore evident that an understanding of non-Newtonian flow and energy transfer may enable substantial economic improvements to be made in a wide diversity of processing techniques.

a. Newtonian Fluids

In Newtonian fluids energy is dissipated by the collision of relatively small molecules. In consequence, Newtonian behaviour is found in all gases, liquids and solutions of low molecular weight. The equation dealing with the relationship between the shearing stress and rate of deformation is termed the rheological equation of state or constitutive equation. The simplest equation of this kind is Newton's law of viscosity (8)

$$\tau_{yx} = -\mu\frac{du}{dy} \tag{3.1}$$

where τ_{yx} is the shearing stress, $\frac{du}{dy}$, shearing rate and μ, a constant called viscosity.

b. Non-Newtonian Fluids

All those fluids for which the flow curve does not follow eqn. (3.1) are said to be non-Newtonian. Those fluids are generally divided into three broad groups.

(1). Time-independent non-Newtonian fluids

(2). Time-dependent non-Newtonian fluids

(3). Viscoelastic fluids.

1. Time-independent Non-Newtonian Fluids

Fig. 3.1 shows different kinds of time-independent non-Newtonian fluids, with the Newtonian fluid included for comparison. τ_y is the yield stress.

Different models for the rheological equation of state have been proposed. Models proposed for fluids with a yield stress are listed in

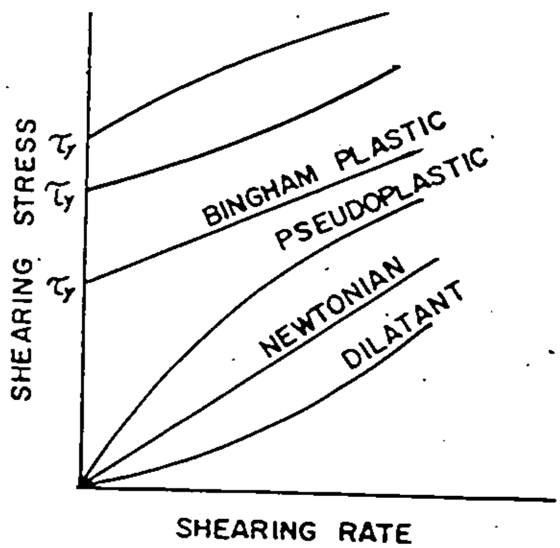


FIG. 3·1 FLOW CURVE FOR TIME-INDEPENDENT FLUIDS

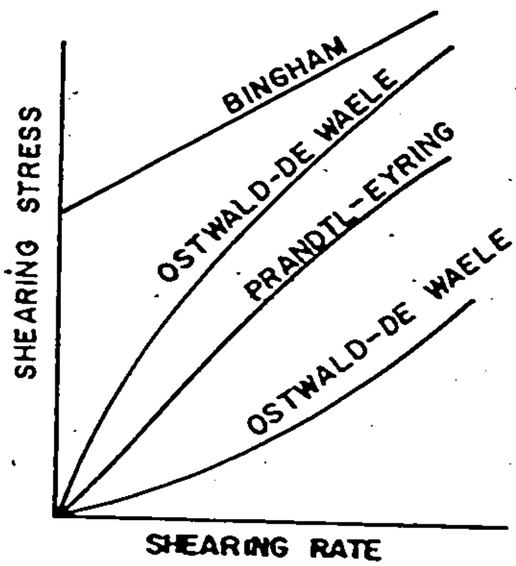


FIG. 3·2 TWO-PARAMETER MODELS

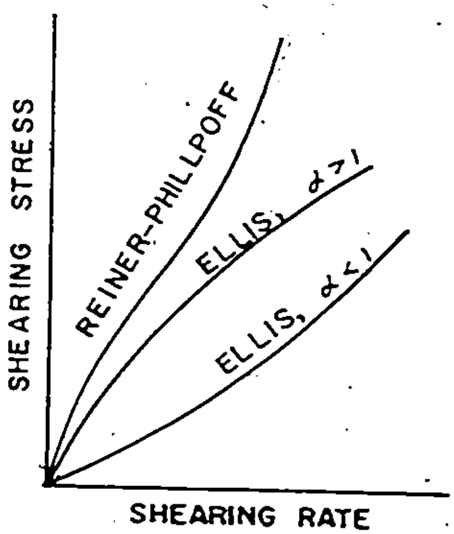


FIG. 3·3 THREE-PARAMETER MODELS

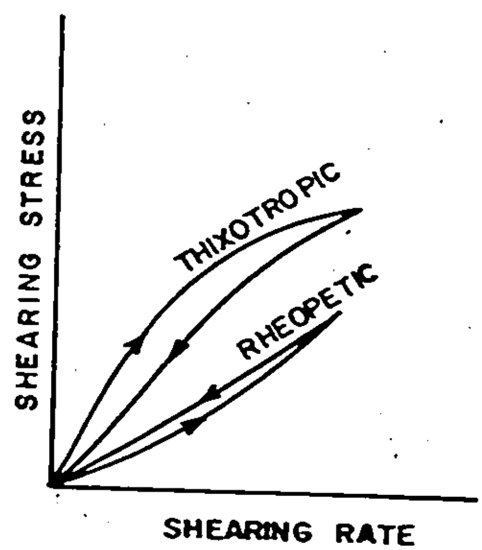


FIG. 3·4 FLOW CURVE FOR TIME-DEPENDENT FLUIDS

Table 3.1 and those without a yield stress in Table 3.2. Some of these models are plotted in Fig. 3.2 and Fig. 3.3.

2. Time-dependent Non-Newtonian Fluids

These fluids are usually classified into two groups, thixotropic and rheopetic fluids (see Fig. 3.4). Their shearing rate is a function of both magnitude and the duration of the shearing stress. They are even possibly dependent upon the time lapse between consecutive applications of shearing stress.

3. Viscoelastic Fluids

These fluids have both viscous and elastic properties. In a purely elastic solid, the shearing stress corresponding to a rate is independent of time, whereas for the viscoelastic, the stress will gradually dissipate, making it time-dependent. Different rheological models have been proposed and detailed in References (54,68,25,4).

Inspite of the efforts expended, the progress made does not yet provide a simple workable model capable of representing all the major non-Newtonian fluids. It is therefore evident that the choice of a proper model will represent one of the central issues in the study of non-Newtonian fluids.

c. Ostwald de Waele Model

Comments are particularly directed at the Ostwald de Waele or power-law model because the theoretical analyses and empirical correlations in all the presently cited references, except (83) on non-Newtonian free convection employ the power-law model. In addition this model is also most commonly used on other fields of research such as momentum and heat transfer in agitated tanks, in laminar and turbulent pipe flow, etc. (31,33,41,56,69).

Table 3.1 Rheological Models with A Yield Stress

Equation	Model	Form**	Empirical Constant *	References
3.2	Bingham plastic	$\tau_{yx}-\tau_y = \frac{\xi}{g_c}\left(\frac{du}{dy}\right)$	τ_y, ξ	7
3.3	I Herschel-Bulkley	$\tau_{yx} - \tau_y = \left[\frac{\xi'}{g_c}\left(\frac{du}{dy}\right)\right]^{1/m}$	m, τ_y, ξ	35,36, 77
3.4	II	$\tau_{yx} - \tau_y = \frac{\xi_0/g_c}{1 + c(\tau_{yx}-\tau_y)^m}\left(\frac{du}{dy}\right)$	c, m, τ_y	
3.5	Crowley-Kitzes	$\tau_{yx} = \frac{\mu_L}{g_c}\left[\frac{1.2+S(\lambda\tau_{yx}^{-.2}+1)^3}{1.2-2S(\tau_{yx}^{-.2}+1)^3}\right]\left(\frac{du}{dy}\right)$	S, μ_L,	17

* Refer to nomenclature for units

** For $\tau_{yx} > \tau_y$; $\frac{du}{dy} = 0$ for $\tau_{yx} < \tau_y$

Table 3.2 Rheological Models Without a Yield Stress

Equation	Model	Form	Empirical Constant *	References
3.6	Power-law or Ostwald de Waele	$\tau_{yx} = \frac{K}{g_c}\left(\frac{du}{dy}\right)^{n-1}\left(\frac{du}{dy}\right)$	K, n	55,62
3.7	Ellis	$\tau_{yx} = \frac{1}{a+b\tau_{yx}^{\alpha-1}}\left(\frac{du}{dy}\right)$	a, b, α	63
3.8	Dehaven	$\tau_{yx} = \frac{\mu_o/g_c}{1+c\tau_{yx}^{m}}\left(\frac{du}{dy}\right)$	c, m, μ_o	19, 20
3.9	Prandtl-Eyring	$\tau_{yx} = a_1 \sinh^{-1}\left[\frac{1}{b_1}\left(\frac{du}{dy}\right)\right]$	a_1, b_1	26,59
3.10	Powell-Eyring	$\tau_{yx} = c_2\left(\frac{du}{dy}\right)+\frac{1}{b_2}\sinh^{-1}\left[\frac{1}{a_2}\left(\frac{du}{dy}\right)\right]$	a_2, b_2, c_2	15
3.11	Reiner-Phillippoff	$\tau_{yx} = \frac{1}{g_c}\left[\mu_\infty + \frac{\mu_o - \mu_\infty}{1+(\tau_{yx}/a_3)^2}\right]\left(\frac{du}{dy}\right)$	a_3, μ_o, μ_∞	57
3.12	Sisko	$\tau_{yx} = a_4\left(\frac{du}{dy}\right) + b_4\left(\frac{du}{dy}\right)^m$	a_4, b_4, m	79

* Refer to nomenclature for units

The parameters K and n in the power-law model

$$\tau_{yx} = \frac{K}{g_c}\left(\frac{du}{dy}\right)^{n-1} \frac{du}{dy} \qquad (3.6)$$

are called the flow consistency and flow behaviour indices respectively. $K\left(\frac{du}{dy}\right)^{n-1}$ is analogous to viscosity in Newtonian fluid and similarly enables quantitative comparison of the consistancy of fluids having the same flow behaviour index. The deviation of n from unity characterizes the extent of departure from Newtonian behaviour.

The power-law model is so far the most extensively used model probably because of the following main properties:

(1). It has relatively few parameters.

(2). The parameters can be easily determined from conventional viscometry measurements.

(3). The rheological data of a large number of substances have been adequately represented by this model over relatively large ranges of shearing rates.

In spite of these advantages of the power-law model, some obvious inadequacies, such as infinite viscosity at zero shearing rate ($n < 1$) and the dependence of one parameter on the other, coupled with the fact that the shearing stress in free convection is thought to be small, have raised some doubt about the use of this model in connection with free convection problems. Tien and Tsuei (83) thus tried to analyse the free convection problem with the Ellis model and believe this model can be used to characterize free convection.

B. <u>Laminar Free Convection Heat Transfer to Newtonian Fluids</u>

Free convection in the laminar boundary layer for Newtonian fluids has been solved for two dimensional objects and bodies with rotational symmetry by Merk and Prins (49) with the approximation method. No detail of the theory but only the final equation for predicting heat flow is given in eqns. (3.13), (3.14) and (3.15).

$$N_{Gr} = \frac{g_c \, \beta \, \rho^2 \, L^3 (T_s - T_\infty)}{\mu^2} \tag{3.13}$$

$$N_{Pr} = \frac{C_p \, \mu}{k} \tag{3.14}$$

$$\overline{N_{Nu}} = C' \, (N_{Gr} \, N_{Pr})^{1/4} \tag{3.15}$$

where C' is a function of Prandtl number and the geometry of the surface of heat source. Table 3.3 gives the C' values of a sphere by the first approximation method. L is equal to the diameter of the sphere.

The analysis of Acrivos on the boundary layer equations also results in equation (3.15) where his parameter is a function of the geometry of the heat source only. His predicted C' is 0.49 with L equal to the radius of the sphere.

Table 3.3 Theoretical values of C'

N_{Pr}	C'
0.7	0.474
1	0.497
10	0.576
100	0.592
∞	0.595

C. Laminar Free Convection Heat Transfer to Non-Newtonian Fluids

a. Acrivos' Theory

The analysis of free convection to non-Newtonian fluids by Acrivos (1) is one of the best reported. The analysis, employing a power-law model to describe the shearing stress and shearing rate relationship, is developed first on a two dimensional system and then extended to the three dimensional axisymmetric case.

1. Formation of Theory

The two dimensional coordinate is shown in Fig. 3.5. The standard boundary layer equation was simplified after assuming the physical properties to be constant except for the density in the buoyancy force term and applying the power-law model in the stress tensor term.

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = g_c \, \beta (T_s - T_\infty)\theta \sin c + \frac{K}{\rho} \frac{\partial}{\partial y} \left[\frac{\partial u}{\partial y}\left(\frac{\partial u}{\partial y}\right)^{n-1}\right] \tag{3.16}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3.17}$$

$$u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 \theta}{\partial y^2} \tag{3.18}$$

where the symbols are explained in the nomenclature. The thermal expansion coefficient is defined by

$$\frac{\rho_\infty}{\rho} = 1 + \beta (T - T_\infty) \tag{3.19}$$

The boundary conditions for equations (3.16) to (3.18) are:

At $y = 0$, $u = 0$, $v = 0$ and $\theta = 1$

At $y = \infty$ and $x = 0$, $u = 0$ and $\theta = 0$

(3.20)

Acrivos defined the following dimensionless N_{Gr}, N_{Pr} and the characteristic velocity, U_c:

$$N_{Gr} = \frac{\rho^2 \, L^{n+2} \, [g_c \, \beta (T_s - T_\infty)]^{2-n}}{K^2} \tag{3.21}$$

$$N_{Pr} = \frac{\rho C_p}{k} \left(\frac{K}{\rho}\right)^{\frac{2}{1+n}} (L)^{\frac{1-n}{1+n}} [L \, \beta \, g_c (T_s - T_\infty)]^{\frac{3(n-1)}{2(1+n)}} \tag{3.22}$$

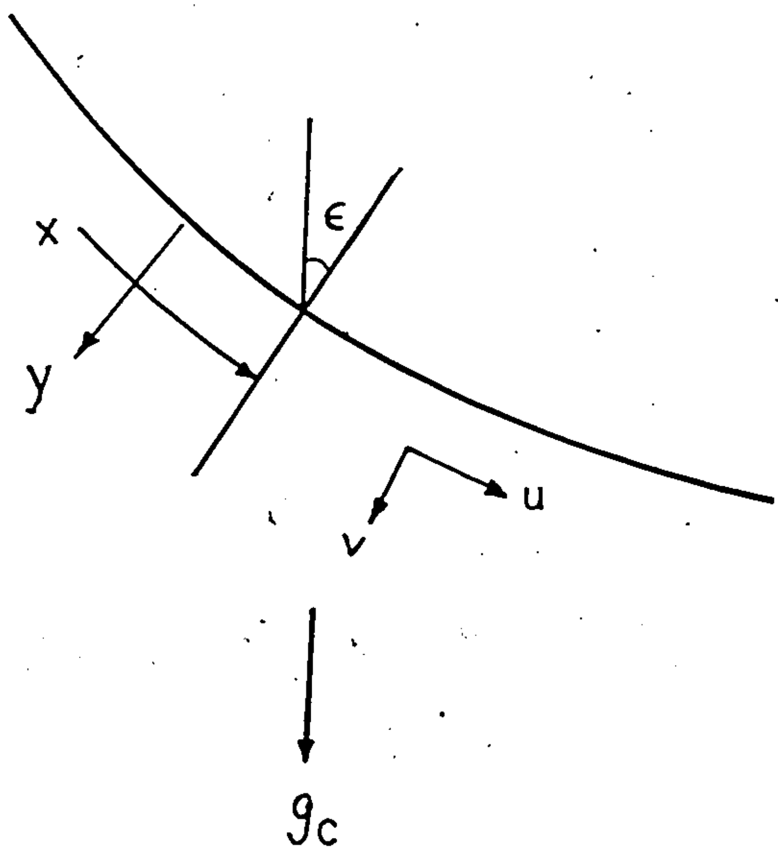


FIG. 3-5 CORDINATE SYSTEM FOR FLOW OVER 2-DIMENSIONAL BODIES

$$U_c = \sqrt{Lg_c\beta(T_s - T_\infty)} \tag{3.23}$$

Having thus defined eqns. (3.21), (3.22) and (3.23), the boundary layer equations can be simplified by a transformation in the position coordinates and the velocity components. Let the new variables be

$$x_1 = \frac{x}{L}, \qquad u_1 = \frac{u}{\sqrt{Lg_c\beta(T_s - T_\infty)}}, \qquad y_1 = \frac{y}{L} N_{Gr}^{\frac{1}{2(n+1)}}$$

$$v_1 = \frac{v}{\sqrt{Lg_c\beta(T_s - T_\infty)}} N_{Gr}^{\frac{1}{2(n+1)}} \tag{3.24}$$

The boundary layer equations then became

$$u_1 \frac{\partial u_1}{\partial x_1} + v_1 \frac{\partial u_1}{\partial y_1} = \theta \sin\varepsilon + \frac{\partial}{\partial y_1}\left[\frac{\partial u_1}{\partial y_1}\left(\frac{\partial u_1}{\partial y_1}\right)^{n-1}\right] \tag{3.25}$$

$$\frac{\partial u_1}{\partial x_1} + \frac{\partial v_1}{\partial y_1} = 0 \tag{3.26}$$

$$u_1 \frac{\partial \theta}{\partial x_1} + v_1 \frac{\partial \theta}{\partial y_1} = \frac{1}{N_{Pr}} \frac{\partial^2 \theta}{\partial y_1^2} \tag{3.27}$$

with the boundary conditions

at $y_1 = 0$, $u_1 = 0$, $v_1 = 0$, $\theta = 1$

at $y_1 = \infty$ and $x_1 = 0$, $u_1 = 0$, $\theta = 0$ (3.28)

Under the asymptotic condition, $N_{Pr} \to \infty$, the appropriate transformation is

$$y_2 \equiv y_1 N_{Pr}^{\frac{n}{3n+1}} = \frac{y}{L} N_{Gr}^{\frac{1}{2(n+1)}} N_{Pr}^{\frac{n}{3n+1}}$$

$$u_2 \equiv u_1 N_{Pr}^{\frac{n+1}{3n+1}} = \frac{u}{\sqrt{Lg_c\beta(T_s - T_\infty)}} N_{Pr}^{\frac{n+1}{3n+1}}$$

$$v_2 = v_1 N_{Pr}^{\frac{2n+1}{3n+1}} = \frac{v}{\sqrt{Lg_c\beta(T_s - T_\infty)}} N_{Gr}^{\frac{1}{2(n+1)}} N_{Pr}^{\frac{2n+1}{3n+1}} \qquad (3.29)$$

Substitute eqn. (3.29) into eqn. (3.25), (3.26) and (3.27), one obtains

$$\frac{\partial}{\partial y_2}\left[\frac{\partial}{\partial y_2}\left(\frac{\partial u_2}{\partial y_2}\right)^{n-1}\right] + \theta \sin\varepsilon = \frac{1}{N_{Pr}^{\frac{2(n+1)}{3n+1}}} \{u_2 \frac{\partial u_2}{\partial x_1} + v_2 \frac{\partial u_2}{\partial y_1}\} \to 0 \qquad (3.30)$$

as $N_{Pr} \to \infty$.

$$\frac{\partial u_2}{\partial x_1} + \frac{\partial v_2}{\partial y_2} = 0 \qquad (3.31)$$

$$\frac{\partial^2\theta}{\partial {y_2}^2} = u_2 \frac{\partial\theta}{\partial x_1} + v_2 \frac{\partial\theta}{\partial y_2} \qquad (3.32)$$

The boundary conditions are the same as in eqn. (3.28) except for one of the conditions at $y = \infty$, which now reads $\frac{\partial u_2}{\partial y_2} = 0$ rather than $u_2 = 0$.

Thus if $\theta = \theta(\eta)$ and

$$u_2 = (\sin\varepsilon)^{\frac{1}{n}}\left[\frac{3n+1}{2n+1}\left(\frac{1}{\sin\varepsilon}\right)^{\frac{3n+1}{n(2n+1)}} \int_0^{x_1} (\sin\varepsilon)^{\frac{1}{2n+1}} dx_1\right]^{\frac{n(n+1)}{3n+1}} f'(\eta)$$

where

$$\eta = \frac{y_2}{\left[\frac{3n+1}{2n+1}\left(\frac{1}{\sin\varepsilon}\right)^{\frac{3n+1}{n(3n+1)}} \int_0^{x_1} (\sin\varepsilon)^{\frac{1}{2n+1}} dx_1\right]^{\frac{n}{3n+1}}} \qquad (3.34)$$

one can obtain

$$\frac{d}{d\eta}(f'')^n + \theta = 0 \qquad (3.35)$$

$$\theta'' + f\theta' = 0 \qquad (3.36)$$

with the boundary conditions

$\theta(0) = 1,\quad \theta(\infty) = 0,\quad f(0) = 0,\quad f'(0) = 0$ and $f''(\infty) = 0.$

The two parameters of particular interest, $f''(0)$ and $\theta'(0)$, can be calculated from

$$-\left(\frac{d\theta}{d\eta}\right)_{\eta=0} = \frac{1}{\int_0^\infty e^{-\int_0^\eta f\,d\eta}\,d\eta}$$

$$\left[\frac{d^2f}{d\eta^2}\right]^n_{\eta=0} = \frac{\int_0^\infty \eta e^{-\int_0^\eta f\,d\eta}\,d\eta}{\int_0^\infty e^{-\int_0^\eta f\,d\eta}\,d\eta} \tag{3.37}$$

In the special case of $n=1$, $\theta'(0)=0.5404$ and $f''(0)=1.08$. The values of $\theta'(0)$ and $f''(0)$ has been shown to be a function of n (see Fig. 2 and 3 in Reference (1)).

By definition, the local Nusselt number is

$$N_{Nu} \equiv -L\left(\frac{\partial\theta}{\partial y}\right)_{y=0} \tag{3.38}$$

This will finally take the form

$$N_{Nu} = -\theta'(0)\left[\frac{2n+1}{3n+1}\right]^{\frac{n}{3n+1}} N_{Gr}^{\frac{1}{2(n+1)}} N_{Pr}^{\frac{n}{3n+1}} \frac{(\sin\epsilon)^{\frac{1}{2n+1}}}{\left[\int_0^{x_1} (\sin\epsilon)^{\frac{1}{2n+1}}\,dx_1\right]^{\frac{n}{3n+1}}} \tag{3.39}$$

after substituting eqns. (3.24), (3.29) and (3.34) into eqn. (3.38). Integration of eqn. (3.39) will give the average Nusselt number.

$$\overline{N_{Nu}} = C\,N_{Gr}^{\frac{1}{2(n+1)}} N_{Pr}^{\frac{n}{3n+1}} \tag{3.40}$$

where C is a function of the surface geometry and n only.

2. Axisymmetric Case

Acrivos used the technique first discovered by Mangler to reduce the boundary layer equations for laminar flow past a surface of revolution to a set of equations identical to those for two dimensional flow. For flow past three dimensional surfaces of revolution, eqns. (3.16) and (3.18) and therefore equations (3.30) and (3.32) still apply. The continuity equation must, however, be modified to

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial y}(rv) = 0 \tag{3.41}$$

where $r(x)$ is the distance of a point on the surface from the axis of symmetry. The following transformation is necessary in order to reduce the three dimensional equations.

$$r_1 = \frac{r(x_1)}{L}, \quad \chi = \int_0^{x_1} r_1(x_1)\,dx_1, \quad \overline{u_2} = u_2 r_1 \tag{3.42}$$

It follows that the extension of equations (3.30), (3.31) and (3.32) to the axisymmetric flow is respectively,

$$\frac{\partial}{\partial y_2}\left(\frac{\partial \bar{u}_2}{\partial y_2}\right)^n + \theta r_1^n \operatorname{Sin}\epsilon\,(\chi) = 0 \tag{3.43}$$

$$\frac{\partial \overline{u}^2}{\partial \chi} + \frac{\partial v2}{\partial y_2} = 0 \tag{3.44}$$

$$\frac{\partial^2\theta}{\partial y_2^2} = \bar{u}_2\frac{\partial\theta}{\partial \chi} + v_2\frac{\partial\theta}{\partial y_2} \tag{3.45}$$

From equations (3.42) and (3.39) the local Nusselt number is

$$N_{Nu} = -\theta'(0)\left[\frac{2n+1}{3n+1}\right]^{\frac{n}{3n+1}} N_{Gr}^{\frac{1}{2(n+1)}} N_{Pr}^{\frac{n}{3n+1}} \frac{(r_1^n \sin\epsilon)^{\frac{1}{2n+1}}}{\left[\int_0^{x_1} r_1^{\frac{3n+1}{2n+1}}(\sin\epsilon)^{\frac{1}{2n+2}}\,dx_1\right]^{\frac{n}{3n+1}}} \tag{3.46}$$

where $\theta'(0)$ has the same function of n as before. Equation (3.46) is the generalization of equation (3.39) for three dimensional axisymmetric flows and is applicable to any surface of revolution.

As a special case which is of great interest to the present study, we consider the natural convection from a sphere. If the characteristic length is taken to be the radius of the sphere, then

$$\sin \varepsilon = \sin x_1 \text{ and } r_1 = \sin x_1 \tag{3.47}$$

Combination of equations (3.46) and (3.47) forms

$$N_{Nu} = -\theta'(0) \left[\frac{2n+1}{3n+1}\right]^{\frac{n}{3n+1}} N_{Gr}^{\frac{1}{2(n+1)}} N_{Pr}^{\frac{n}{3n+1}} \frac{(\sin x_1)^{\frac{n+1}{2n+1}}}{\left[\int_0^{x_1} (\sin x_1)^{\frac{3n+2}{2n+1}} dx_1\right]^{\frac{n}{3n+1}}} \tag{3.48}$$

The average Nusselt number is again represented by equation (3.40). In the particular case of $n = 1$, equation (3.40) becomes

$$\overline{N_{Nu}} = C[\ N_{Gr}N_{Pr}\ ]^{1/4} \tag{3.49}$$

which is the same as equation (3.15). The value of C for a sphere with L equal to the radius of the sphere are shown in Table 3.4.

It is important to remember that Table 3.4 is based on constant physical properties. This simplification might introduce non-negligible errors in the C value if the physical properties of a solution are strong functions of temperature. Three other major assumptions made are:

Table 3.4

Theoretical Value of C for Sphere

n	c
1/10	0.44
1/2	0.45
1	0.49
3/2	0.52

(1) The boundary layer equations must be applicable. It is well known from boundary layer theory that the boundary layer theory is applicable only when the transfer of momentum and energy occurs in a very thin region near the surface. It therefore is necessary that

$$N_{Gr}^{\frac{1}{2(n+1)}} N_{Pr}^{\frac{n}{3n+1}} >> 1$$

(2) The modified Prandtl number must be large so that the inertia term in equation (3.30) is negligible. It has been proven (1) for Newtonian fluids that equation (3.30) is fairly accurate for $N_{Pr} \geq 10$.

(3) The frictional heat generation term in the energy equation is negligible.

b. Approach by Empirical Method

Because of the many assumptions made in obtaining equations (3.15) and (3.49) for Newtonian fluids and equation (3.40) for non-Newtonian fluids, the validity of the theoretical equations would certainly be questionable when applied to data obtained from real situations. But both equations (3.15) and (3.40) are certainly valid in the form of

$$\overline{N_{Nu}} = C_1 (N_{Gr} N'_{Pr})^{C_2} \tag{3.50}$$

C_1 and C_2 are evaluated from experimental data.

Tien and St. Pierre (81) defined the modified Grashof and Prandtl numbers using a dimensional analysis approach as:

$$N_{Gr} = \frac{g\beta\Delta T L^{[(n+2)/(2-n)]}}{(K/\rho)^{\frac{2}{2-n}}} \tag{3.21}$$

$$N'_{Pr} = \frac{C_p \rho}{k} \left[\frac{K}{\rho} \right]^{\frac{1}{2-n}} L^{\frac{2(n-1)}{n-2}} \tag{3.51}$$

Reilly (61) correlated his experimental data with equation (3.50) (except he used $N_{pr}'^{n}$ instead of N_{pr}'), (3.21) and (3.51) to a high degree of accuracy. Sharma (78) found that Acrivos' equation failed to correlate his data satisfactorily and claimed that equations (3.21), (3.50) and (3.51) correlated both his and Reilly's data on Carbopol.

IV EXPERIMENTAL

A. Equipment

a. Physical Properties Measurement

1. Density and Thermal Expansion Coefficient

The densities of all the polymer solutions were determined with the use of a 25 ml nominal size Gay Lussac pycnometer with a capillary stopper, and a constant temperature bath whose temperature can be maintained to 0.1°C. From the density data, the thermal expansion coefficient of each test fluid was determined with the help of eqn. (4.1)

$$\beta = -\frac{1}{\rho}\left(\frac{\partial\rho}{\partial T}\right)_P \qquad (4.1)$$

The density and thermal conductivity data of water as a function of temperature were obtained from known sources (30).

2. Thermal Conductivity

The thermal conductivity of polymer solutions was determined using a concentric cylindrical thermal conductivity cell designed by Riedel (65). This design was used by St. Pierre (81), Reilly (61) and Sharma (78). Its design and construction are fully described in Appendix III.

3. Rheological Properties

The shearing rates encountered in laminar free convection heat transfer are usually of the order of 0.1 to 1 sec^{-1} (1). It is therefore desirable to employ a viscometer capable of measuring the rheological properties in this range. For the present study a coaxial cylindrical viscometer, manufactured by Brookfield Engineering Laboratories was used. The L.V.T. model employed has eight speeds, ranging from 0.3 to 60 rpm. This permits a shearing rate ratio of 200:1. This viscometer is equipped with a U.L. adapter for low viscosity measurements. The U.L. adapter consists of a precision cylindrical spindle rotating inside an accurately machined stainless steel tube, whose inside diameter is 1.087 inches and length is about five and one-quarter inches. In the present study, all viscosity measurements were performed with the U.L. adapter because of the method used to evaluate the shearing stress and shearing rates (81, 43).

Three spindles of different length were used to measure the rheological properties of all the test fluids, except water.

Spindle No. 0 : A cylindrical bob with an outside diameter equal to 0.990 inch and a length equal to 3.575 inches

Spindle No. 1 : A spindle with an outside diameter equal to 0.740 inch and a length equal to 2.656 inches

Spindle No. 2 : A spindle with an outside diameter equal to 0.737 inch and a length equal to 0.508 inch.

The lowest shearing rates obtainable from this viscometer are 0.5 Sec^{-1}, 0.2 sec^{-1} and 0.1 sec^{-1} corresponding to spindle No. 0, 1 and 2, respectively. The viscometer is guaranteed to be accurate to within 1% of the full scale value and to have a reproducibility of 0.2%. Although this viscometer seems to yield fairly good results, it has certain limitations which will be discussed in the " Calibration " section.

b. Heat Transfer Measurements

1. Heat Transfer Sphere

The five inch diameter copper test sphere consisted of two copper hemispheres with a spherical transite core and a teflon mounting tube. Two copper hemispheres were cast by an external contractor and then machined precisely in the Central Research Shop, University of Windsor, to two hemispheres with outside diameters equal to 5 inches and wall thickness equal to 1/4 inch. The equipment used to machine the hemisphere was designed and built by the Central Research Shop. Unfortunately, due to a defect in the casting, there were tiny air holes in the sphere thus causing electrical shorting. All attempts made to plug the air holes by copper plating, both chemically and electrically were unsuccessul. These two hemispheres finally had to be abandoned and two more constructed from a commercial cold rolled bar of pure copper. The machined hemispheres fitted snugly together by mating grooves in each hemisphere (see Fig. 4.1). A rubber O-ring ensured the junction to be waterproof. Six 4/40 set screws equally spaced and three 1/8 inch O.D. brass locking pins were used to hold the two hemispheres in position. Six mica supports, three on each hemisphere were properly built into the inner surface of the hemispheres to support the transite core centrally. The protrusion of the supports was 0.045". A one inch diameter threaded opening was cut at the north pole of one hemisphere and served as the outlet for thermocouple wire leads and power lines.

Five AWG 36 copper-constantan thermocouples were embedded into the hemispheres to measure the outer surface temperature of the sphere. These thermocuples were made from HFD-36-T thermocouple wire manufactured by Thermo Electric Limited. The constanton wire was covered and insulated by a compound with the trade name "Krypton". The copper wire lay on the outside of the Krypton insulation of constanton wire and was covered again by another layer of insulation. This forms a coaxial cylindrical structure with the constantan wire in the centre and copper wire in the annulus. The outside diameter of the thermocouple wire was about 0.04 inch. This type of thermocouple wire was selected for the three following reasons. Firstly, the metal wires were small so that heat conduction through them was minimized. Secondly, the insulation material could withstand higher temperatures compared to other types of organic insulation material. Thirdly, the structure of the insulation was suitable for point junction welding when fabricating

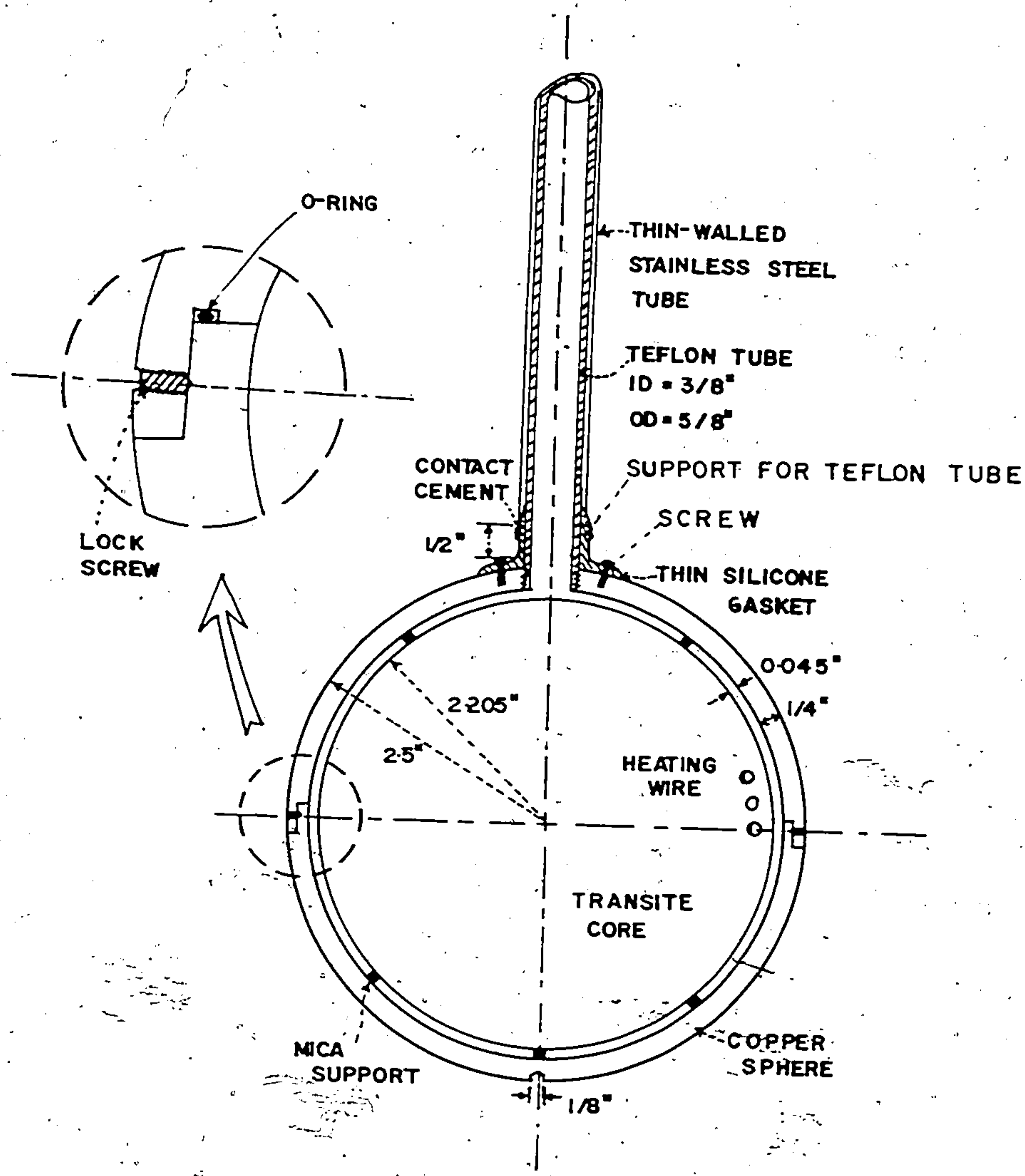


FIG 4·1 HEAT TRANSFER SPHERE

the thermocouple junction. The thermocouples were made in the following way: A portion of the outside insulation is first removed, whereas a comparatively shorter portion of the inner insulation is removed. The bare section of the copper wire provided a contact point in forming an electric circuit while welding. The junction of the thermocouple had to be very close to the inner insulation so that the only contact point was the probe itself.

The mounting of thermocouples into the copper wall has a significant effect on the accuracy of the surface temperature measurement. Several ways of installing the thermocouples were tried. The first method involved cementing the thermocouple into a hole drilled through the centre of a screw, leaving the thermocouple probe protruded from the end of the screw. The screw was then placed into a threaded hole on the copper wall until the probe reached the bottom and was in good contact with the copper wall. A thermocouple installed this way does not measure the true surface temperature(see Appen. VI). Another disadvantage of this method was that the thermocouple usually broke because of the tension it received when tightly screwed into contact with the copper wall. A second method suggested used a plastic-aluminum compound to hold the thermocouple. When this method was tested on a flat plate, as shown in Appen. VI, it was found that the temperature measured was substantially higher than the true temperature. Finally, a method which was accurate to within 0.2% was found. This involved soldering the thermocouple probe into a brass pin (see Fig. 4.2) which was forced into a hole drilled into the copper wall and ensured that the thermocouple probe was in good contact with the copper sphere. Since the pin had to be snugly fitted into the hole, it was very important to keep the outside of the pin absolutely clean of the solder while soldering

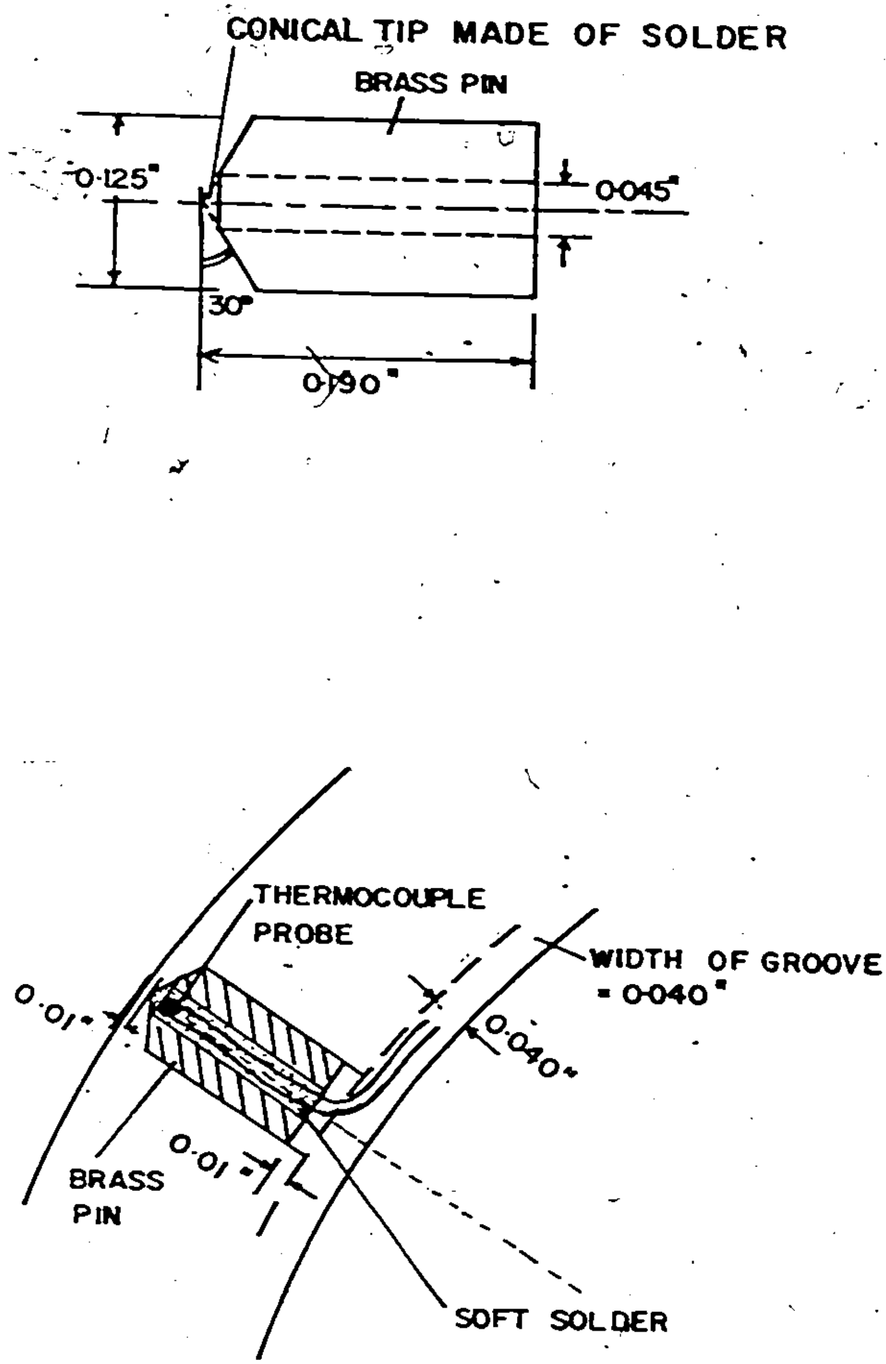


FIG. 4·2 INSTALLATION OF THERMOCOUPLES

the thermocouple into the pin. The pin was first wrapped with Teflon tape except for the right hand side open end, illustrated in Fig. 4.2. The pin was then held upright with the open end facing upward and heated with a soldering gun. After the pin became hot enough, solder would melt into the hole until it was almost filled. The thermocouple was then inserted into the bottom of the pin hole immediately after the removal of the soldering gun. The excess of solder would flow out. Since the pin is wrapped with teflon tape, no solder could come into contact with the outer surface of the pin. The thermocouple was held in position until the solder solidified. After the pin cooled down, the teflon wrap was removed. A tiny cone had to be made at the bottom end of the pin so that the pin would match exactly the shape and dimension of the hole in the copper wall. This was easily done by putting a small lump of solder on to the end of the pin and filing the solder into the right shape and size.

The exact position of the five thermocouples are shown in Fig. 4.3. Tiny grooves of depth 0.05" were cut vertically from each thermocouple location and led upward as far as possible. The grooves cut in the lower hemisphere did not extend to the upper hemisphere because of installation problems. After the thermocouples were put into positions, each leading wire was glued into the corresponding groove using Pliobond adhesive, (mfg. by Goodyear Tire and Rubber Company). The embedding of the lead wire would reduce the conduction effect to the thermocouple.

The 2.205" radius spherical core was machined from a piece of 5" X 5" rectangular transite bar. Transite is a compound of cement with asbestos fill-in. Its machinability is excellent. A total of 40 equally spaced grooves at 0.156" intervals from each other were cut laterally as

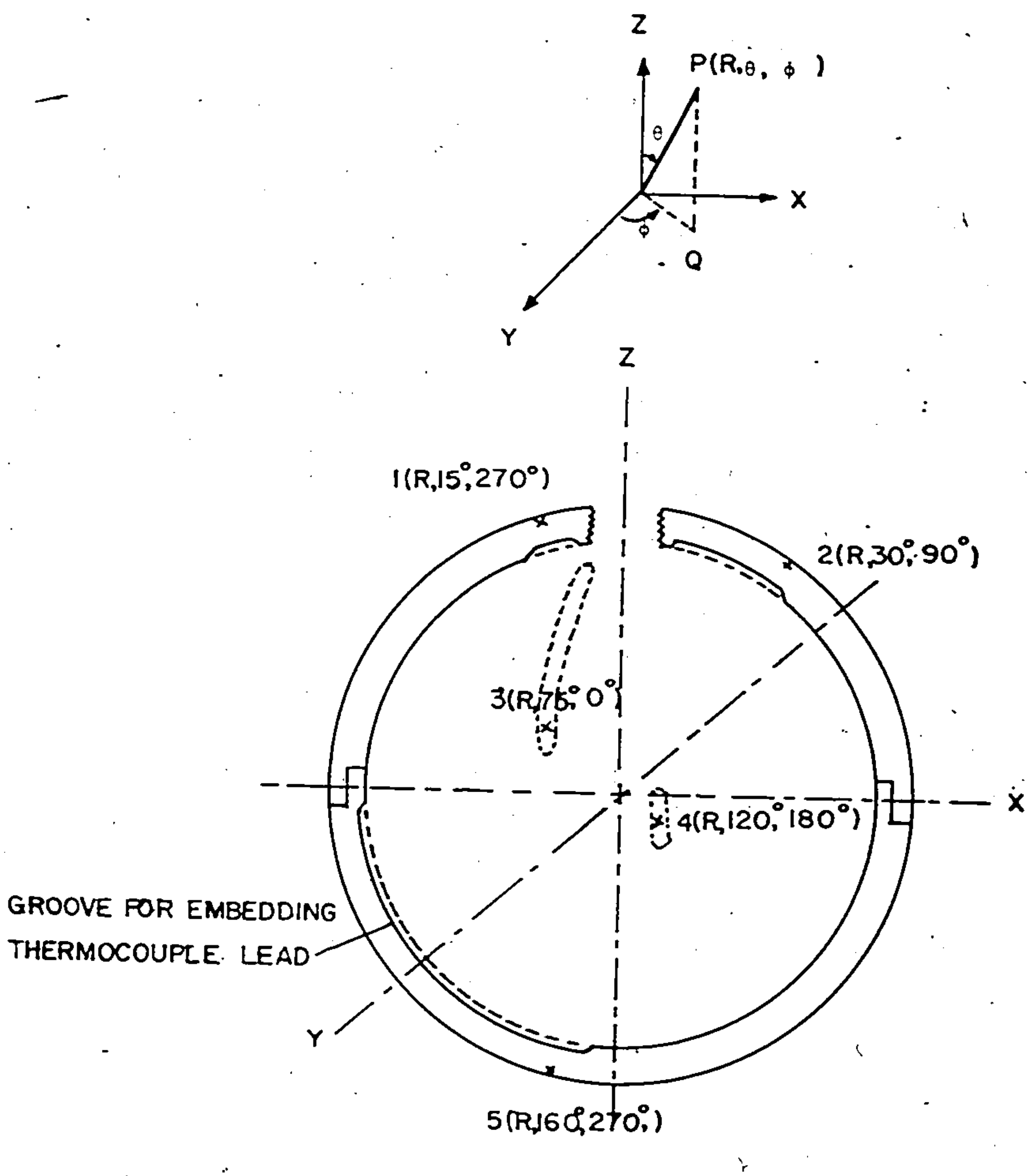


FIG 4·3 LOCATION OF THERMOCOUPLES IN SPHERE shown in Fig. 4.4. All the grooves with angles $\leq 45.9°$ [Fig. 4.5] were cut perpendicular to the surface. In the region $45.9 < \phi < 90$, the cuts were slightly tilted so that when the heating wires were wound into the grooves, the wire would not slip out of the grooves. These grooves were cut in such a way that the centre of the heating wire, when lying on the bottom of each groove, would be equally spaced from each other and 2.123" away from the centre of the core. The cuts on the lower half of the core were symmetrical with those on the upper half. Heating wire, type B. & S. 20 Chromel C from Hoskins Manufacturing Co. of Detroit was continually wound into the groove from north to south poles. The vertical groove leading from north to south pole enables the heating wire to step from one groove into the next groove (see Fig. 4.4). The total resistance of the heater thus formed was about 20 ohms. The two ends of the heating element were connected to a size 16 power line with the help of two embedded set screws. The power line that joins the heating element at the South pole had to pass through the centre hole of the core. The insulation of the power line from the south pole to about half an inch above the North pole was ceramic beads. A fiberglass insulated AWG 24 copper-constantan thermocouple was placed at the centre hole to record the temperature of the core. The grooves as well as the two poles where the connecting screws sat were then filled with Sauereisen cement filler so that both the heating element and power line were insulated (see Fig. 4.6). The core was then heated in an oven at 110°C for a few days to drive out the water in the core. This procedure was necessary because if the core was not dry, water would evaporate from the core when it was heated, and this water vapour would come into contact with the comparatively cooler copper wall and condense. Power could then leak when enough condensate accumulated.

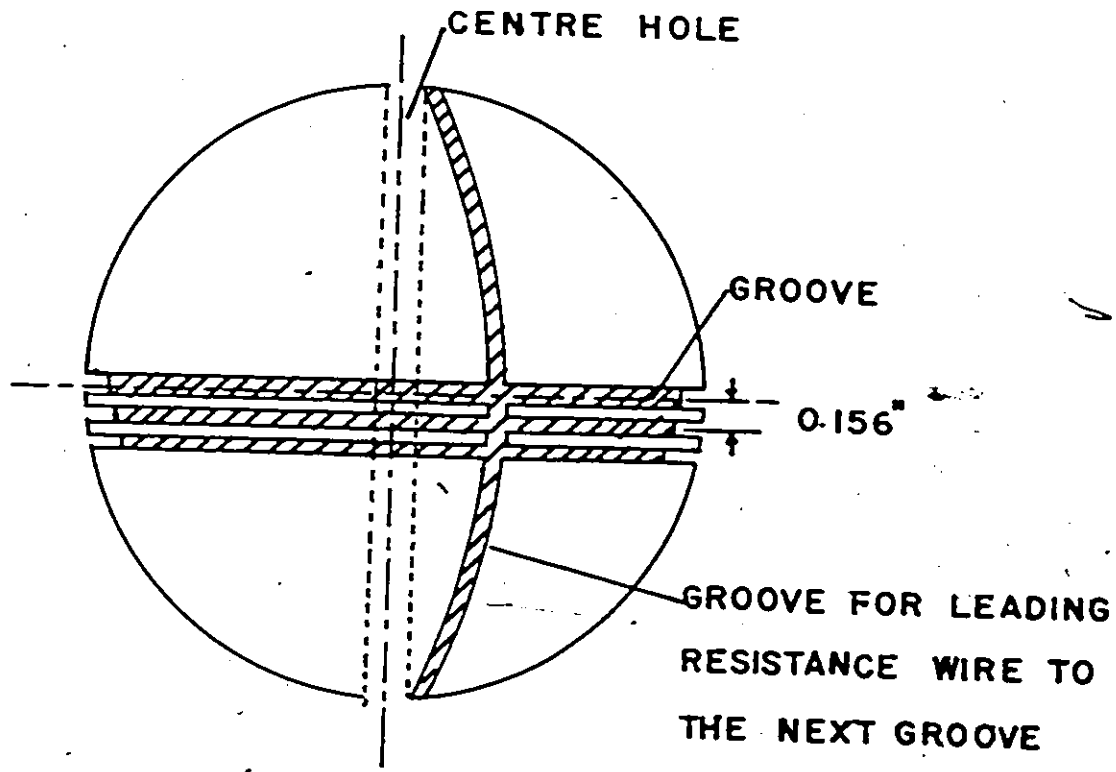
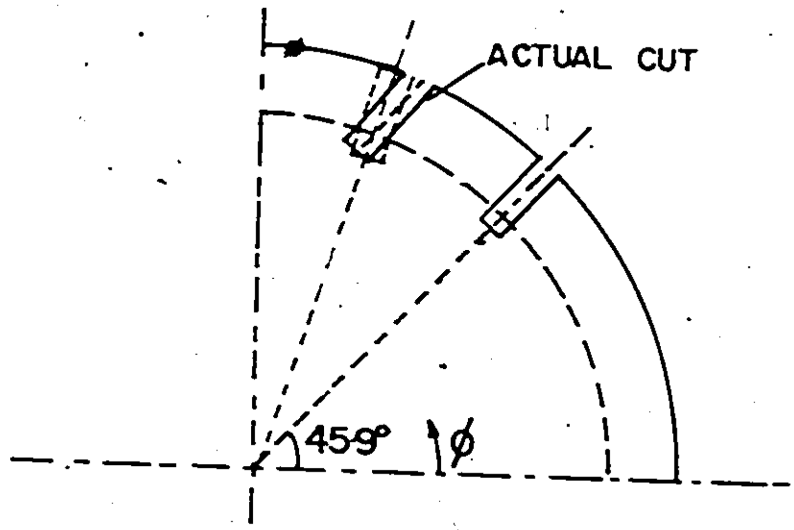


FIG-4.4 GROOVES IN TRANSITE CORE



FIG· 4·5 ACTUAL CUTS OF GROOVE NEAR THE POLES.

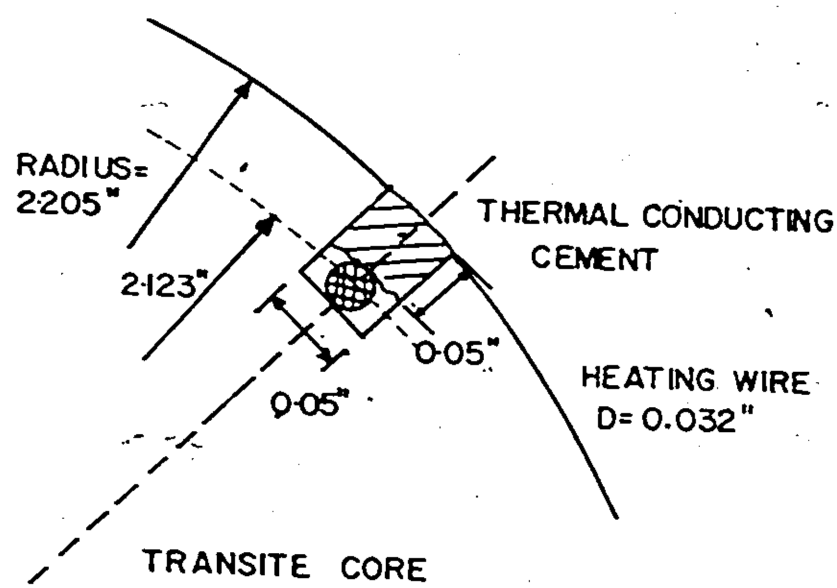


FIG. 4·6 DETAILS OF GROOVE

The two hemispheres and the core were then ready to be put together. The core was first set in the lower hemisphere. The upper hemisphere was then lowered to join the lower half. Care was taken to ensure that the thermocouple leads from the lower hemisphere did not bend, twist or entangle while lowering the upper hemisphere. Any mishaps would either break the thermocouple or prevent the two hemispheres from closing up completely. After the hemispheres were put together and locked by six set screws and three brass pins, the two foot long Teflon tube, with the thermocouple wires and power line passing through its centre was screwed into the threaded hole at the north pole of the sphere to give the construction details as shown in Fig. 4.1. The sphere was supported by a Nylon stand and the centre of the sphere was 6.5" from the bottom of the tank and 15.5" from the free surface of the solution.

2. Heat Transfer Tank

All the heat transfer experiments were conducted in a cubical glass tank 24" X 24" X 24" with a capacity of about 60 gallons. The tank was fabricated from 1/4 inch glass sheet on four sides. Aquarium cement was used to seal the glass sheets to the angle iron.

3. Mixing Tank and Stirrer

A 200 gallon capacity polyethylene tank was used for two purposes. Firstly, it was used to prepare the polymer solution employing a 1/4 h.p. Lightnin mixer. Secondly, deaeration of water and polymer solutions took place in this tank.

4. Temperature Measurement System

In addition to the six thermocouples built into the sphere, there was a movable thermocouple mounted on a precision two-way micrometer. This micrometer, which was capable of measuring accurately to 0.001 inch, had a horizontal range of 4" and a vertical range of 5.5". This movable thermocouple was built to measure the temperature profile of the boundary layer. There was another portable thermocouple used to measure the bulk temperature of the fluid All the eight thermocouple leads were led to a rotary switch which served to select the particular thermocouple and connected the circuit to the reference junction and the measuring device (see Fig. 4.7). The measuring device was either a Leeds and Northrup precision potentiometer or a Barber-Colman mV recorder.

5. Power Supply and Its Measurement Equipment

The normal 115 volt AC supply was used to power the heater inside the sphere. The schematic flow chart of the power supply and associated measurements is shown in Fig. 4.7. The voltage was stabilized by a voltage regulator model ITLR-1000, manufactured by Perkin Electronic Company. This regulator has an accuracy of 0.2% for line changes from 105-125 volts and a response time of 0.05 to 0.1 second. The main switch was furnished with a 10 ampere fuse and thus provided adequate protection to the system. The Variac made by American Superior Electric Company had a twelve ampere capacity and was used to control the heat input to the sphere. Weston model 904 multirange (75,150 and 300 V) voltmeter and ammeter both of which are accurate to 0.5% of full scale, were used to measure the voltage and current respectively. The by-pass switch was used for current measurement. The highest voltage and current used in this study was about 110 volts and 5 amperes.

B. Calibration

a. Physical Properties Measurement

1. Pycnometer

The pycnometer was calibrated by using deaerated distilled water. The temperature of the fluid inside the pycnometer as well as the volume of the pycnometer was calibrated as a function of bath temperature. The temperature of the fluid in the pycnometer was measured by a calibrated AWG 40

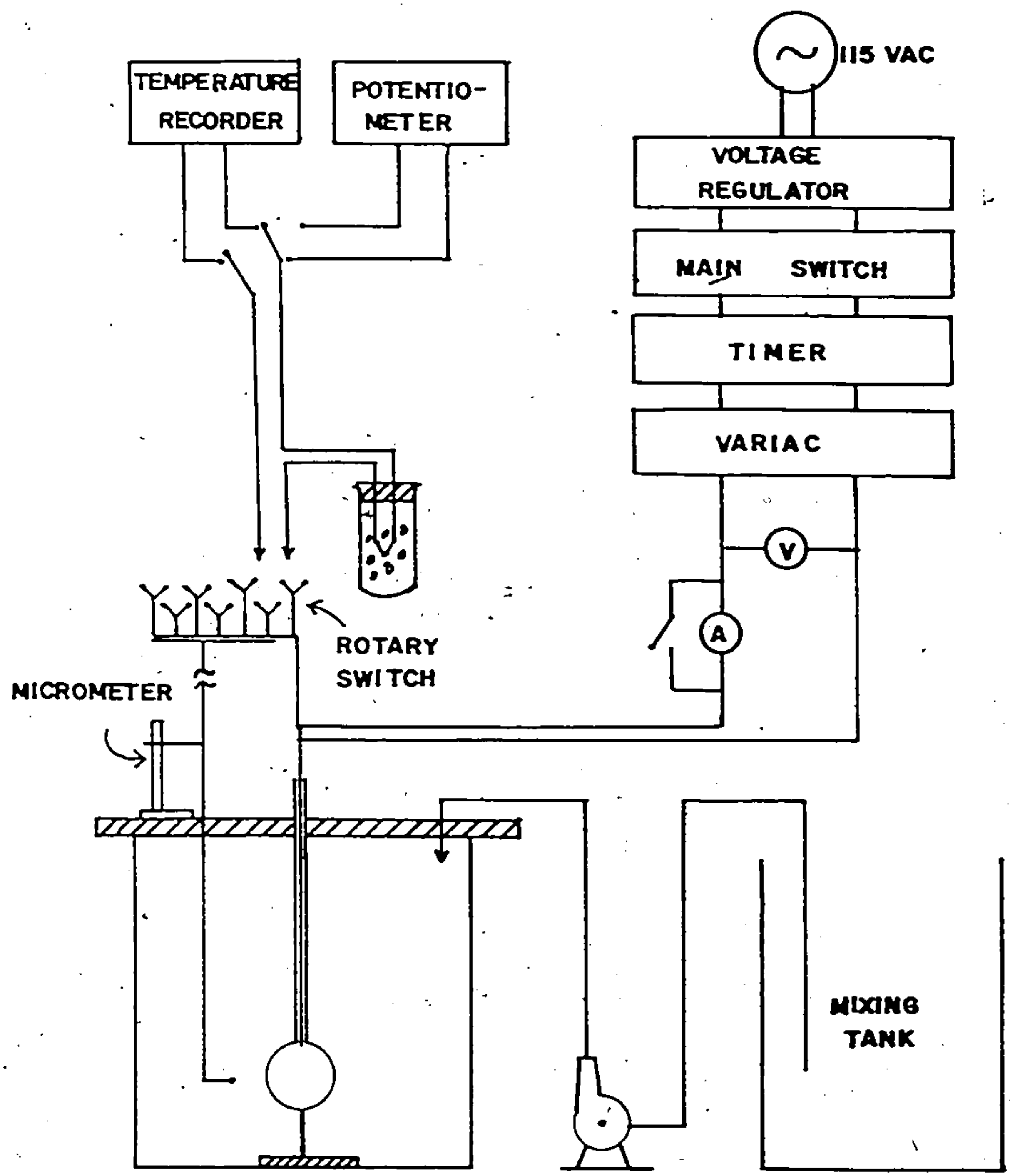


FIG. 4·7 FLOW CHART OF ARRANGEMENT OF EXPERIMENTAL EQUIPMENT copper-constantan thermocouple. The temperature difference between the pycnometer and water bath is shown in Fig. A2.1 and Table A2.1 in Appen. II. [Note: All figure and table numbers with a prefix "A" refer to Appendix]. The temperature difference exists due to the fact that heat loss by conduction occurs through part of the pycnometer which is exposed to air at a different temperature. The temperature correction was made for the calibration and all other density measurements. The volume of the pycnometer was computed using the weight of water in the pycnometer and the appropriate density in Ref. (30). The results are represented by eq. (4.2) and Fig. A2.2 as a function of bath temperature.

$$V\ [\mathrm{ml}] = 24.1103 + 3.1901 \times 10^{-4}\ T\ [°F] \tag{4.2}$$

2. Thermal Conductivity Apparatus

As shown in Appendix III , the governing equation for the apparatus is:

$$q/\Delta T = A + B\ k \tag{III.2}$$

where A and B are equipment constants. These constants are functions of temperature as well as certain parameters such as the heat flux through the thermal conductivity cell. The average of the three thermocouples used to measure the heating surface temperature of the fluid was found to register the potential 0.0029 mV lower than the tabulated values provided by Omega Engineering Incorporated, while the average of the six thermocouples for measuring the temperature of the cooling surface was 0.0039 mV too low. The total heat input q was determined from the voltage and current, which in turn were measured by a Heathkit V-7A AC voltmeter and a Weston model 904 ammeter respectively. These meters were calibrated by the Electronic Research Centre of the University of Windsor and are accurate to 0.5% full scale value.

The liquids used to evaluate the cell constants A and B were redistilled water and 20% and 40% ethylene glycol in water. The $q/\Delta T$, where ΔT is the temperature difference between the heating and cooling temperature of the thermal conductivity cell, vs. average temperature plot for water 20% and 40% ethylene glycol and 10% sucrose in water are given in Fig. A3.3 and listed from Table A3.1 to A3.4. The average temperature is the arithmetic mean of the heating and cooling temperatures. The data points clearly form curves rather than straight lines. Since all the experiments performed in this study were from room temperature to about 150°F, straight lines were assumed within this temperature range. The following equations were obtained by the least square method:

For water,

$$\frac{q}{\Delta T} = 1.7047 - 4.0433 \times 10^{-3}\ T \tag{4.3}$$

For 20% ethylene glycol solution,

$$\frac{q}{\Delta T} = 1.5342 - 3.6002 \times 10^{-3}\ T \tag{4.4}$$

Mean error = 1.3% .

For 40% ethylene glycol solution,

$$\frac{q}{\Delta T} = 1.3431 - 2.8374 \times 10^{-3}\ T \tag{4.5}$$

Mean error 1.3% .

For 10% sucrose solution,

$$\frac{q}{\Delta T} = 1.6262 - 3.6884 \times 10^{-3}\ T \tag{4.6}$$

Mean error 1.1% .

Reliable data on water, 20% and 40% ethylene glycol (64) were used to evaluate A and B of equation (III.2) at a temperature interval of 10°F. The results are listed in Table A3.5 and plotted in Fig. A3.4. For computational convenience, A and B were fitted into second order polynomials.

$$A = 0.4328 + 8.9560 \times 10^{-4} T - 2.7413 \times 10^{-6} T^2 \quad (4.7)$$

$$B = 3.9521 - 1.9424 \times 10^{-2} T + 2.2417 \times 10^{-5} T^2 \quad (4.8)$$

A and B predicted from equations (4.7) and (4.8) along with $q/\Delta T$ from equations (4.3), (4.4) and (4.5) were used to obtain Table A3.6. The mean errors for water, 20% and 40% ethylene glycol are 0.39, 0.78 and 0.13% respectively.

Thermal conductivity measurements on 10% sucrose solution (64) were used to check the accuracy of the apparatus and method of evaluation. Table A3.7 shows the results and the mean error was 0.87%. Reported thermal conductivity of liquids with an error of 2% were rated excellent (70, 71, 38). It therefore appears that reliable and precise thermal conductivity data can be obtained employing the relationship: $q/\Delta T$ vs. T as established with enough data points.

3. Viscometer

For all the non-Newtonian polymer solutions, the shearing stress and rate have to be determined in order to evaluate the rheological properties. The method of Krieger and Maron (43), developed to evaluate the flow curve of non-Newtonian fluids from concentric viscometer data, is based on the differential equations obtained by Krieger and Elrod (42), for the case in which the outer cylinder rotates and the inner cylinder is stationary. St. Pierre (81) has shown that the same differential equations can be obtained for the opposite case, ie. having a stationary cup and a rotating bob as the Brookfield Synchro-Reitric Viscometer. Thus the rheological properties of the non-Newtonian fluids were evaluated by the "single bob" method whose details are given in Appendix IV.

This method required the use of an equivalent length instead of the actual length of the rotating bob in order to correct for the end effects of the apparatus. The details of evaluating the equivalent length, shearing stress and rate are given in Appendix IV. The equivalent lengths of the three different spindles were obtained with Newtonian fluids, ie. distilled water, certified ethylene glycol and three types of standard oil from the Cannon Instrument Company. Each spindle was calibrated by at least two fluids at different temperatures and various (bob) rotational speeds. Sample volumes for spindle no. 0, no. 1 and no. 2 were 20, 45 and 42 ml. respectively.

The equivalent length of spindle 0 behaves somewhat abnormally below a dial reading of 40% (see Fig. A4.2). The equivalent length increased sharply as the dial reading decreased. It therefore is obvious that less accurate data is obtained at the lower dial reading. Fig. A4.2 was used to interpolate the equivalent length for all runs. With Fig. A4.2 which has a mean error of 1.8%, the viscosity of water at 68 °F was measured as 2.30 lbm/ft.hr., 5% lower than the reported value (30). The actual length of spindle no. 0 is 3.573 inches. Fig. A4.3 and A4.4 show that the equivalent length for spindle no. 1 and no. 2 are not dependent on the dial reading of the viscometer. The average eqivalent lengths for spindle no.1 and no. 2 were 2.845 inches and 0.512 inch compared with the actual length of 2.565 inches and 0.27 inch.

b. Heading Transfer Measurement

1. Thermocouple

The five thermocouples used to measure the surface temperature of the sphere were calibrated in position by immersing the sphere into a constant temperature bath. A precision grade thermometer calibrated to 0.1 °C was used as the secondary standard. The potentials recorded for the thermocouples were compared to the tabulated value and the difference represented by:

$$\Delta mV = mV_{mea.} - mV_{tab.} \qquad (4.9)$$

$$\Delta mV = -2.8\text{x}10^{-3} + 2.8\text{x}10^{-3}\ mV_{mea.} \qquad (4.10)$$

The portable thermocouple which was used to measure the bulk temperature of the fluid was also calibrated:

$$\Delta mV = -7.8\text{x}10^{-3} + 1.3\text{x}10^{-3}\ mV_{mea.} \qquad (4.11)$$

The units of ΔmV, mV_{mea} and mV_{tab} are given in millivolts.

2. Ammeter, Voltmeter and Potentiometer

As mentioned previously, the ammeter and voltmeter were calibrated by the Electronic Research and Design Shop of the University of Windsor and found to be accurate to within 0.5% of the full scale value. The precision potentiometer used for emf measurement was accurate to 0.05% of the full scale value.

C. Procedure

a. Preparation of Test Solution

Details regarding the preparation of test solutions are given in Appendix I.

b. Cleaning of Sphere

Before each series of runs, the surface of the copper sphere was carefully cleaned with a metal polish. This cleaning and shining of the sphere was done to insure minimal effects due to surface conditions.

c. Physical Properties Measurement

1. Density

The solution whose density was to be measured was first deaerated under vacuum. While filling the pycnometer with the air-free solution, care was taken to prevent any air bubbles from being trapped inside the pycnometer. The pycnometer was then immersed in a constant temperature bath to the calibration depth. When the temperature of the fluid in the pycnometer became constant i.e. when fluid no longer overflowed from the capillary stopper, the excessive fluid on the outlet of the stopper was carefully removed with a filter paper. The outside surface of the pycnometer was dried using a soft tissue before weighing. The bottle was dried again, especially at the junction of the stopper and the bottle, and reweighed. The average weight was used for density calculations. At each temperature, the density measurement was repeated twice. The density of the solution at that particular temperature was taken as the average of the three measurements.

2. Thermal Conductivity

The procedure for thermal conductivity measurement is discussed in detail in Appendix III.

3. Rheological Properties

The rheological properties probably are the most important physical properties that affect the flow condition. Since the viscosity of polymer solutions is usually unstable, it is necessary to measure the rheological properties of the solution and the heat transfer data simultaneously.

The following procedures were followed in measuring the rheological properties.

a. The solution was deaerated.

b. A proper spindle was connected to the viscometer.

c. The proper amount of sample was pipetted into the cup. (Refer to "Calibration" section for sample volume)

d. The cup was hooked to the lower part of the viscometer so that the spindle was immersed with no air bubbles adhering to the spindle.

e. The cup of the viscometer was immersed in the preset constant temperature bath.

f. The viscometer was leveled.

g. A proper speed was selected and the force noted at steady-state conditions.

Steady state was usually obtained after one minute, but for more viscous solutions, it was not unusual to take one hour to reach steady state. At least three readings were taken for each speed at a given temperature and the above repeated for all speeds that gave readings within the dial range.

With all the dial readings at different speeds, the method of Krieger and Maron (43) was employed to determine the shearing stress and rate. The details are given in Appendix IV.

d. Heat Transfer Measurement

The following steps were followed in obtaining heat transfer data.

a. The clean sphere was mounted in position as shown in Fig. 4.7.

b. Then a freshly prepared solution was pumped from the mixing tank into the heat transfer tank.

c. Next, the tank was covered with a thin plastic sheet to reduce evaporation losses.

d. The solution was allowed to stand for several minutes.

e. Energy was supplied to the system heaters and the temperature of the heating surface and the bulk fluid recorded when steady state was achieved.

f. Finally the voltage and current readings were noted, and

g. Steps (e) to (f) repeated at different heat input level.

The time required to obtain steady state was dependent on the heat input to the sphere. At lower heat fluxes, it usually took about an hour but at higher heat fluxes, as much as several hours were required. Steady state was assumed to have been attained when the temperature change was less than 2 microvolts in a 15 minute period.

V. RESULTS

A. Physical Properties

a. Density and Thermal Expansion Coefficient

The densities of all polymer solutions were measured and expressed in polynomial form as a function of temperature. The thermal expansion coefficient was evaluated from the basic definition of Thermodynamics ,

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P \qquad (4.1)$$

rather than the approximation method suggested by equation (3.19). The measured densities and calculated thermal expansion coefficients are listed in Tables A2.3 to A2.6 in Appendix II and graphically illustrated in Figures 5.1 to 5.8. As one would expect the density of the solution increased as the amount of dissolved polymer increased. The density of each solution was clearly distinguishable as a function of concentration. The thermal expansion coefficient of each solution, however, was very similar to that of water.

The polynomial used to express the density and thermal expansion coefficient, as well as other physical properties, is in the form of equation (5.1)

$$Y = k_0 + k_1T + k_2T^2 + k_3T^3 + \ldots\ldots \qquad (5.1)$$

The polynomial coefficients and mean error of each correlation for density and thermal expansion coefficient were given in Table 5.1 and 5.2. The thirteen solutions used were also numbered in Table 5.1, in order to facilitate later references to the solution.

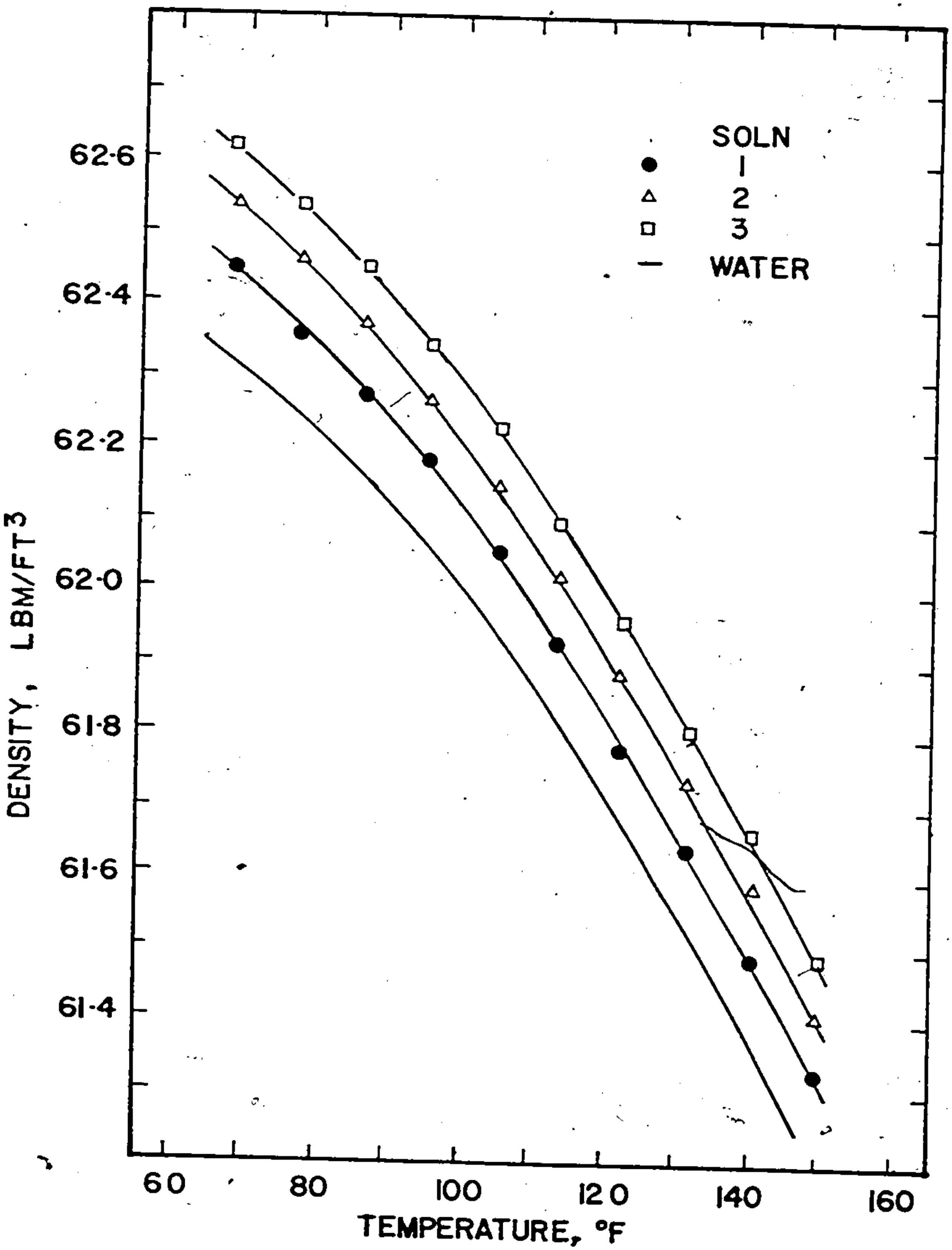


FIG. 5·1 DENSITIES FOR CARBOPOL 934 SOLUTIONS

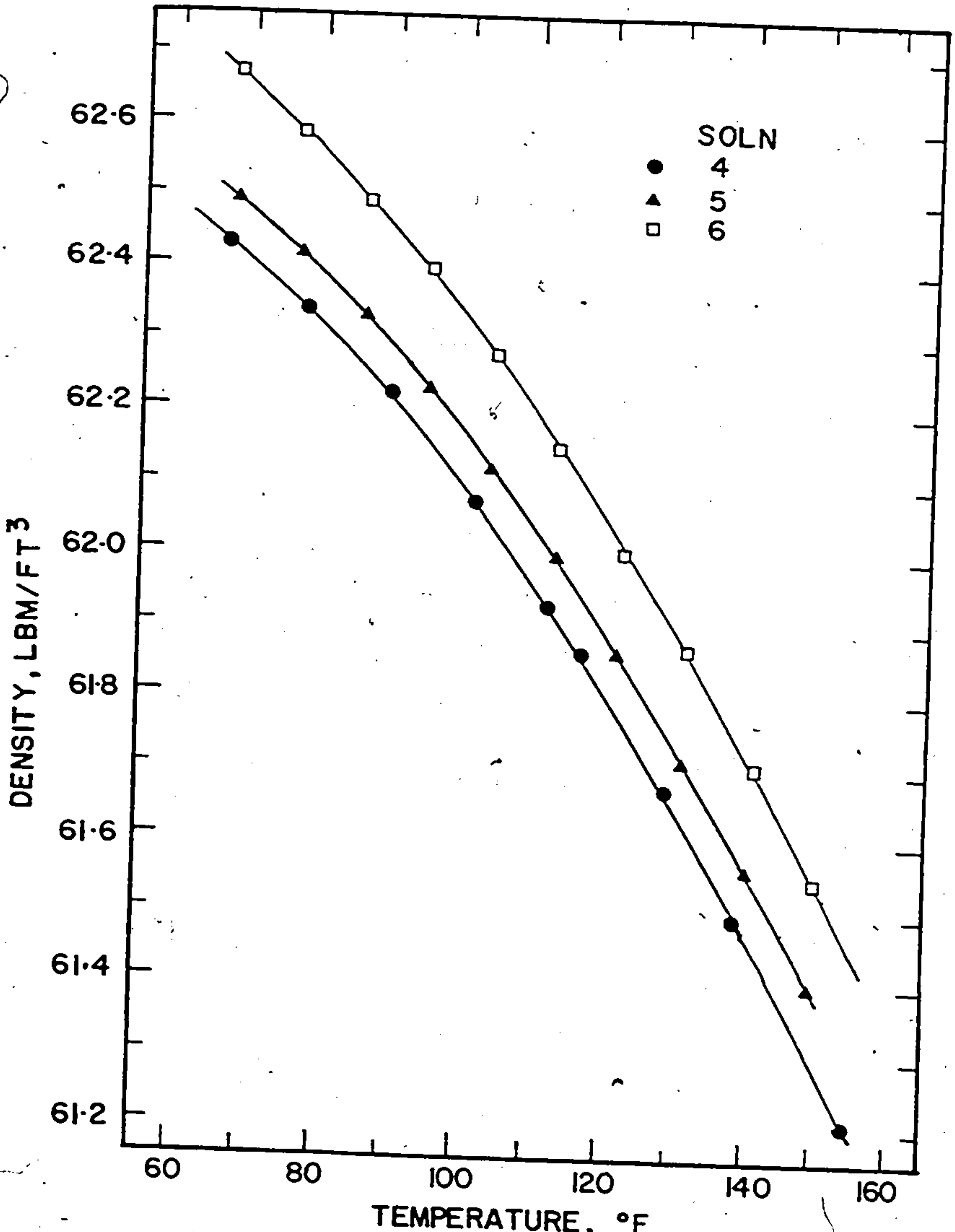


FIG. 52 DENSITIES FOR CARBOSE SOLNS.

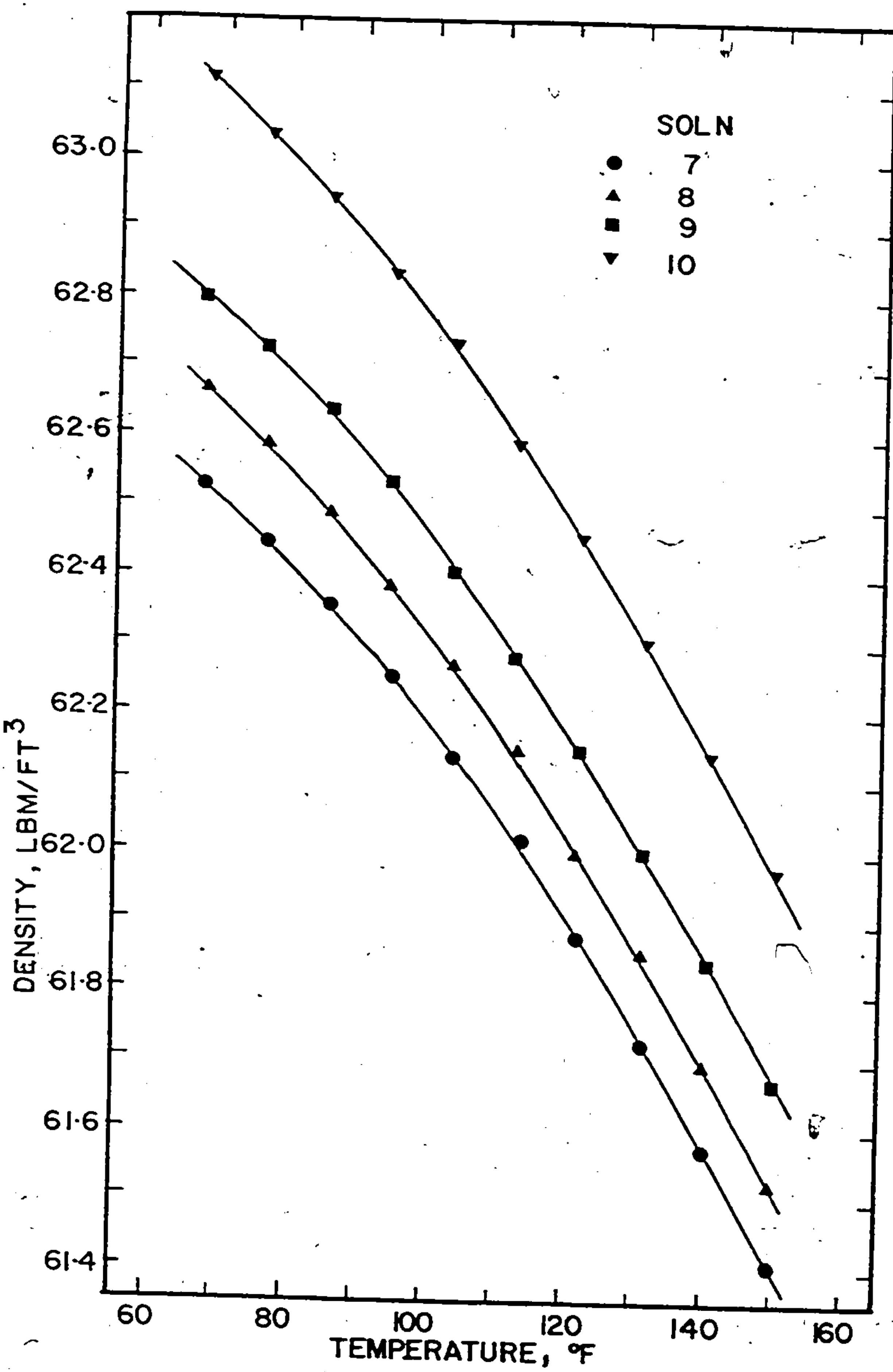


FIG. 5·3 DENSITIES FOR CARBOSE SOLNS.

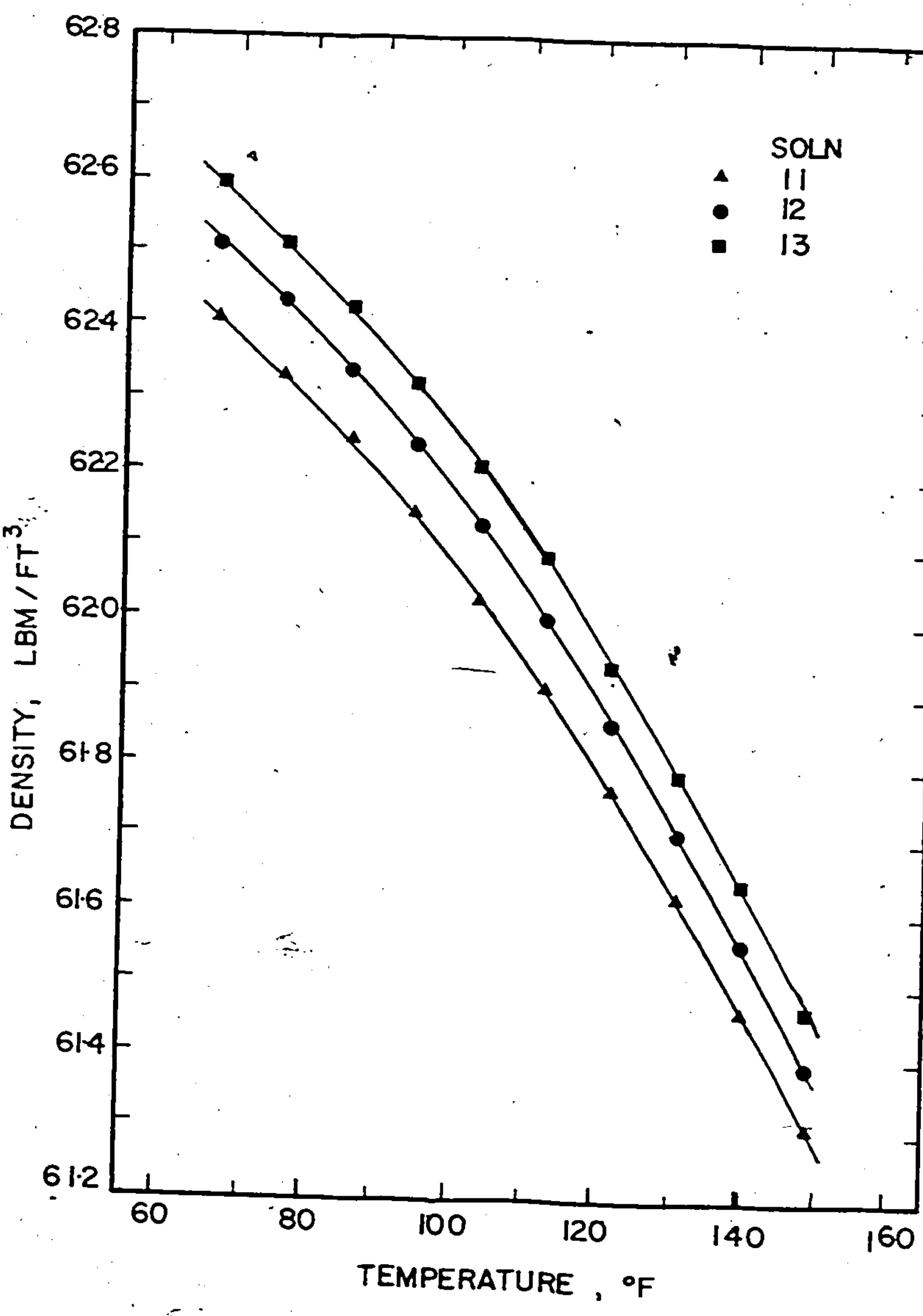


FIG.5·4 DENSITIES FOR NATROSOL 250 H SOLNS.

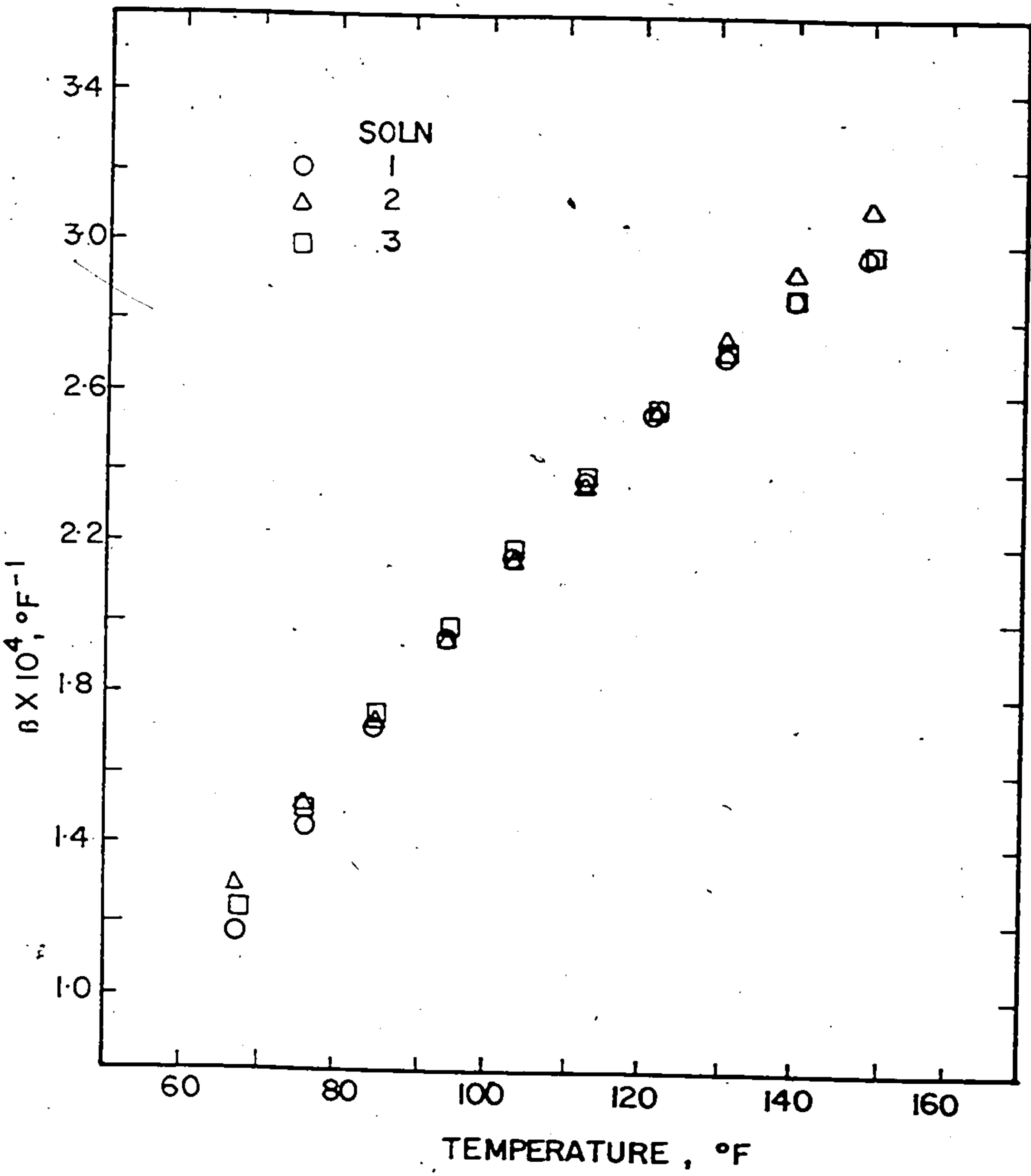


FIG. 5.5 THERMAL EXPANSION COEFFICIENTS FOR CARBOPOL 934 SOLNS.

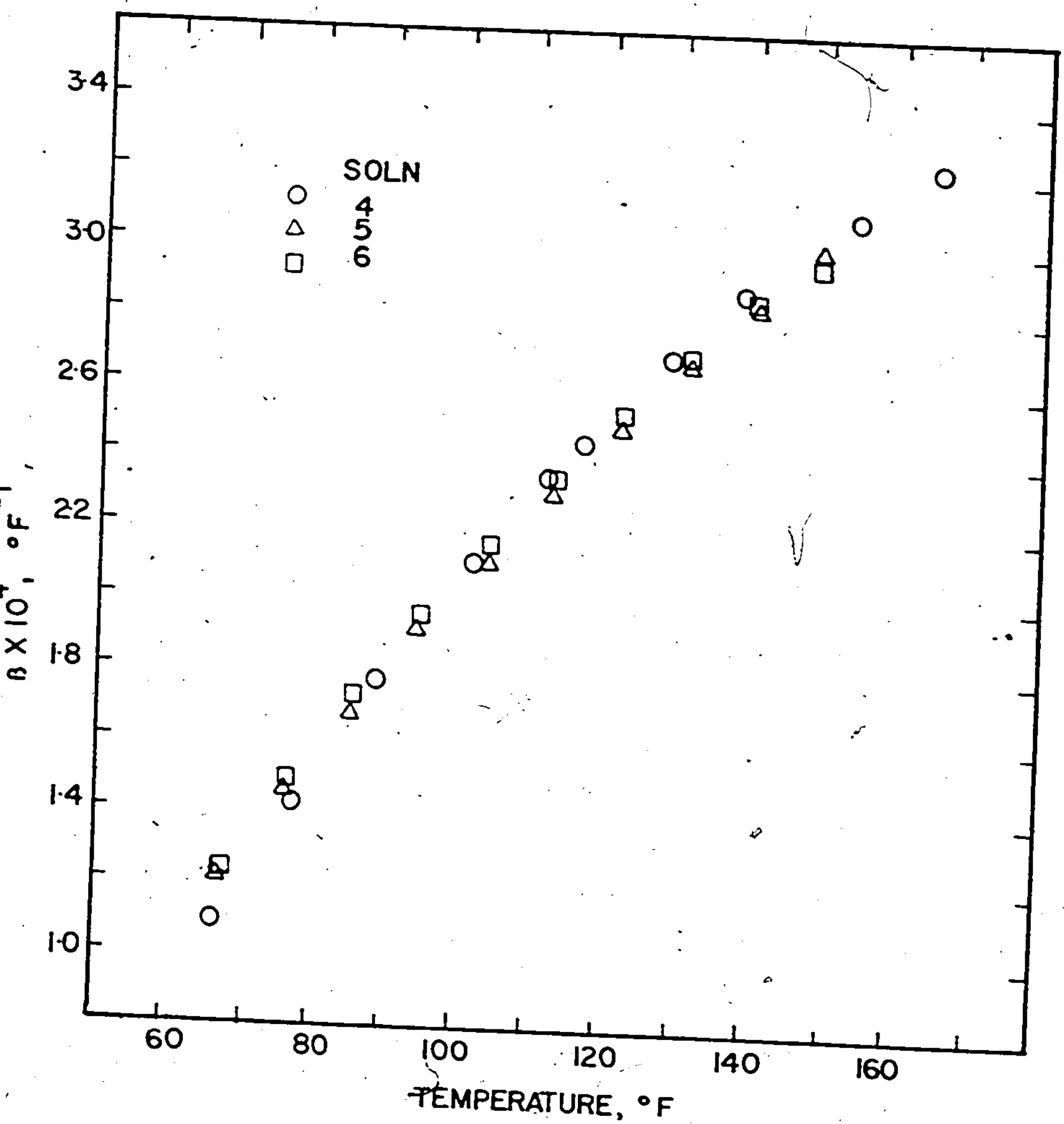


FIG. 5·6 THERMAL EXPANSION COEFFICIENTS FOR CARBOSE TM SOLNS.

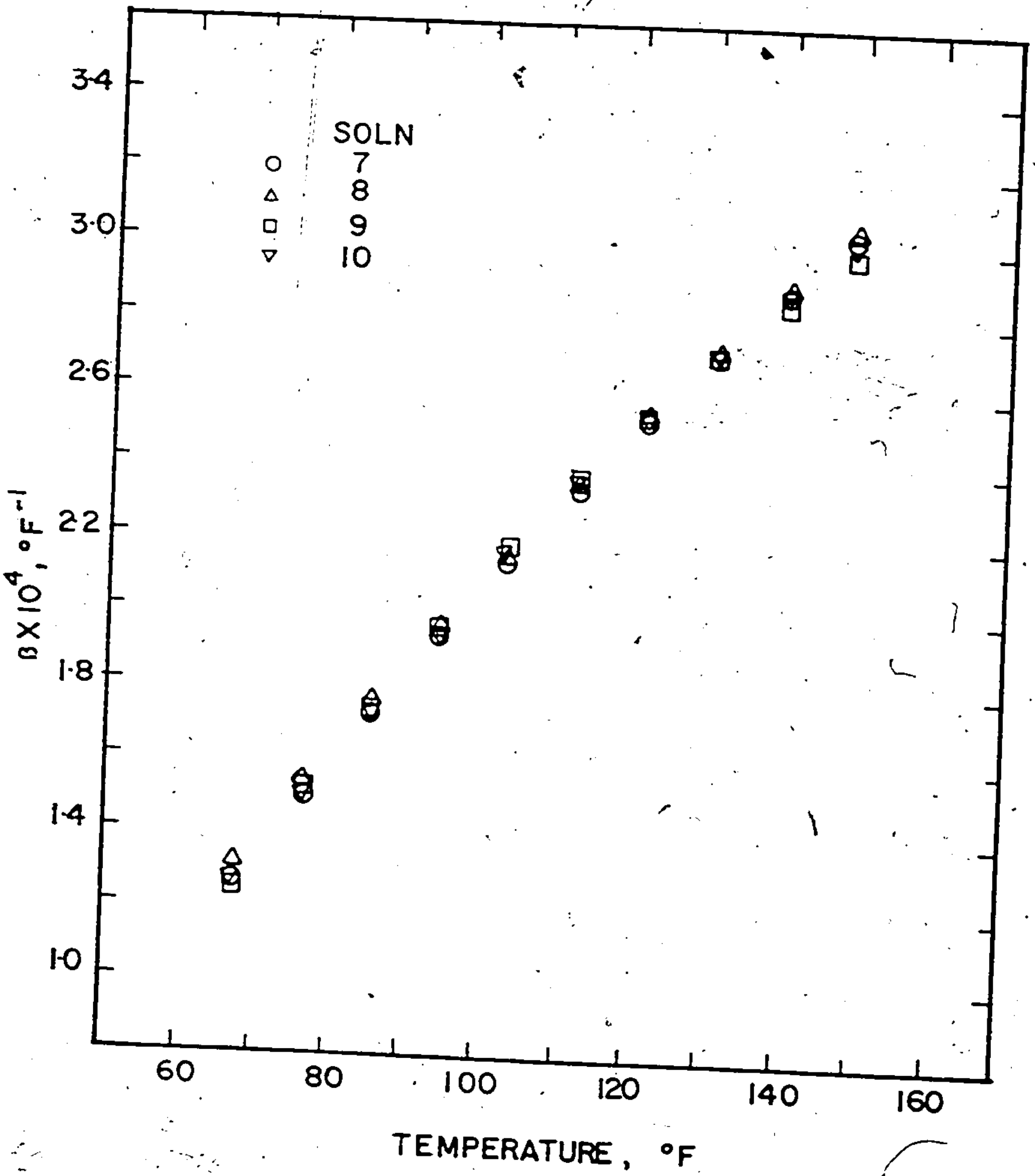


FIG. 5·7 THERMAL EXPANSION COEFFICIENTS FOR CARBOSE 1M SOLNS.

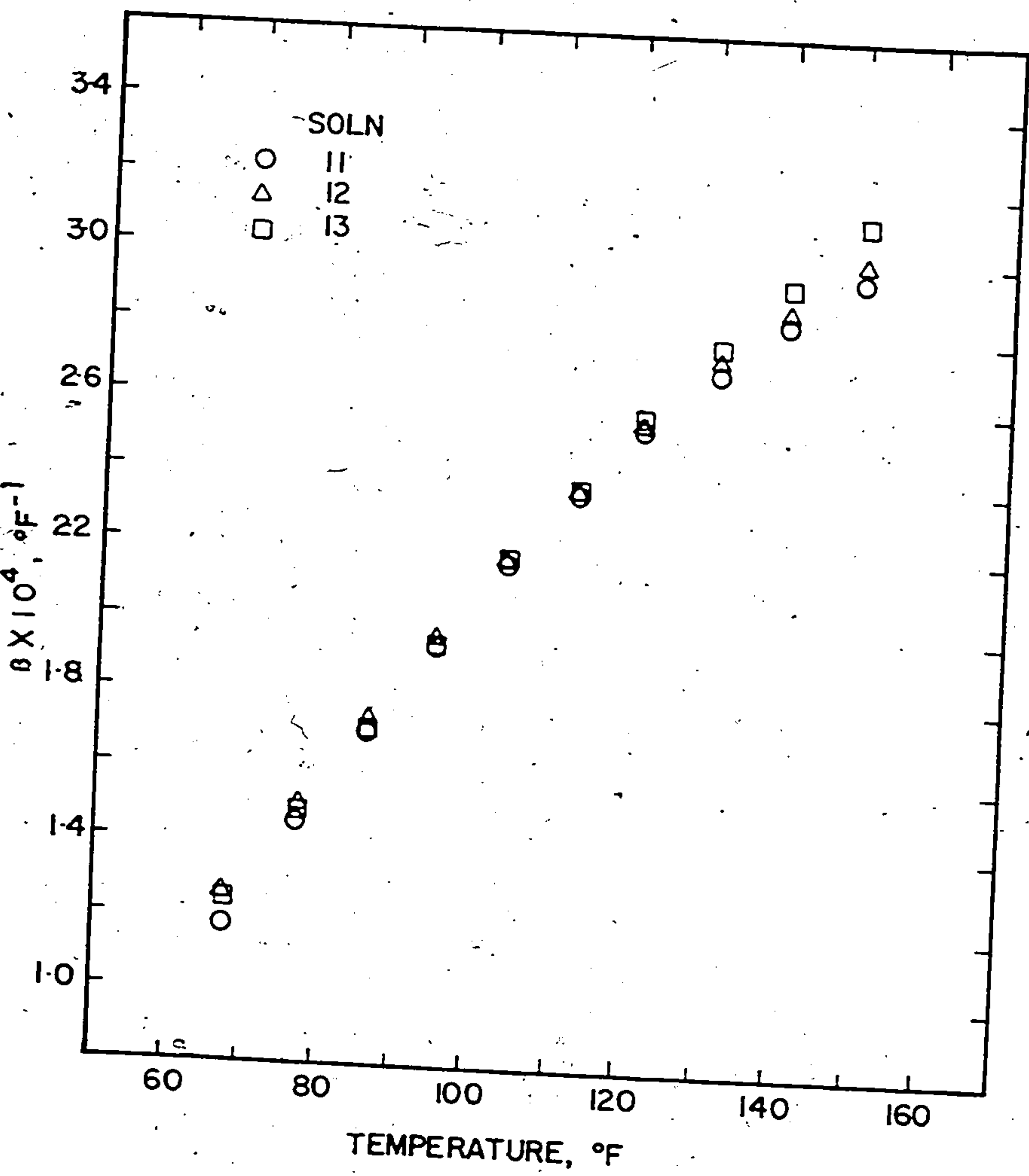


FIG. 5·8 THERMAL EXPANSION COEFFICIENTS FOR NATROSOL 250H SOLNS.

TABLE 5.1

POLYNOMIAL COEFFICIENTS FOR DENSITY

SOLUTION	NO.*	POLYNOMIAL COEFFICIENTS k_0	$k_1 \times 10^{3}$	$-k_2 \times 10^{4}$	$k_3 \times 10^{7}$	MEAN ERROR $\times 10^{3}$
0.5 CARBOPOL 934	1	62.398	9.382	1.543	2.692	5
1.0 CARBOPOL 934	2	62.610	2.977	0.985	0.951	5
1.5 CARBOPOL 934	3	62.521	8.549	1.457	2.481	4
0.25 CARBOSE IM	4	62.335	10.540	1.562	2.018	4
0.5 CARBOSE IM	5	62.563	6.076	1.161	2.235	2
1.0 CARBOSE IM	6	62.683	7.863	1.374	1.113	5
0.5 CARBOSE IM	7	62.828	3.920	1.011	1.113	3
1.0 CARBOSE IM	8	62.828	3.920	1.011	1.113	3
1.5 CARBOSE IM	9	62.975	7.737	1.375	2.228	5
2.5 CARBOSE IM	10	63.206	6.173	1.222	1.710	4
0.5 NATROSOL 250H	11	62.360	9.731	1.523	2.634	4
0.75 NATROSOL 250H	12	62.364	7.042	1.301	1.997	4
1.25 NATROSOL 250H	13	62.677	5.796	1.134	1.343	2

* Numbers were used to represent the solutions. Solution 7 to 13 contained 0.1% Sodium Benzoate. Refer to Appendix I for more detail.

TABLE 5.2

POLYNOMIAL COEFFICIENTS FOR THERMAL EXPANSION COEFFICIENT

SOLN NO.	POLYNOMIAL COEFF. $-k_0 \times 10^{4}$	$k_1 \times 10^{6}$	$-k_2 \times 10^{8}$	MEAN ERROR $\times 10^{2}$
1	1.568	4.879	1.238	1
2	0.632	3.136	0.468	4
3	1.331	4.621	1.143	7
4	1.671	4.937	1.197	1
5	9.495	3.636	6.698	1
6	1.222	4.320	1.010	1
7	7.721	3.409	0.566	1
8	5.853	3.117	0.453	4
9	1.221	4.398	1.001	1
10	1.546	4.817	1.207	1
11	1.546	4.817	1.207	1
12	1.114	4.092	8.953	2
13	0.899	3.537	0.572	1

b. Thermal Conductivity

The experimental results of $q/\Delta T$ vs T for all the 13 solutions are listed in Tables A3.8 to A3.11 and plotted in Figures A3.5 to A3.7. These plots show that the thermal conductivity of the polymer solutions were slightly lower than that of water and the difference between different concentrations is hardly significant, at least statistically. As a matter of fact, it was even hard to justify on statistical grounds that the three correlations in Figures A3.5, A3.6 and A3.7 were different, but assuming that each different polymer might very well have a distinct effect of its own on the thermal conductivity of water, it was decided to treat the data in three groups.

For Carbopol 93 4 solutions

$$\frac{q}{\Delta T} = 1.7273 - 4.3843 \times 10^{-3}\, T \qquad (5.2)$$

For Carbose IM solutions,

$$\frac{q}{\Delta T} = 1.7121 - 4.3439 \times 10^{-3}\, T \qquad (5.3)$$

For Natrosol 250 H solutions,

$$\frac{q}{\Delta T} = 1.7074 - 4.0644 \times 10^{-3}\, T \qquad (5.4)$$

The mean errors and standard deviations of slope and intercept of equations (5.2) to (5.4) are listed in Table 5.3. The thermal conductivities of these solutions as a function of temperature are shown in Figure 5.9 and Table A3.12.

c. Rheological Properties

The rheological properties were evaluated with extra care. Every dial reading used to determine the shearing stress and shearing rate is the average of at least three measurements. The shearing stress and shearing rate of the thirteen pseudoplastic solutions (see Fig. A4.5 in

TABLE 5.3

ERRORS FOR $q/\Delta T$ CORRELATIONS

SOLUTIONS	MEAN ERROR	STANDARD DEVIATION	
	%	INTERCEPT	SLOPE.*10^3
CARBOPOL 934	1.46	0.0254	0.2324
CARBOSE 1M	1.11	0.0095	0.0846
NATROSOL 250H	1.21	0.0177	0.1543

TABLE 5.4

POLYNOMIAL COEFFICIENTS FOR K

SOLN	POLYNOMIAL COEFFICIENTS				MEAN ERROR *10 %
	k_0	k_1	k_2	k_3	
1	2.3846E-4	-1.9533E-5	5.5415E-9		1.69
2	-1.1982E-3	4.4883E-5	-2.9434E-7	5.0786E-10	0.33
3	-8.6693E-3	2.0715E-4	-3.6227E-7		[illegible]
4	3.4690E-4	-6.6570E-5	4.8580E-8	-1.1962E-10	4.42
5	7.2582E-4	-1.1491E-5	7.5664E-8	-1.8217E-10	0.93
6	2.6784E-3	-3.2220E-5	1.0744E-7		3.86
7	7.4628E-4	-1.1624E-5	6.7710E-8		3.19
8	3.1351E-3	-3.2482E-5	3.2957E-7	-7.3306E-10	1.28
9	1.3461E-2	-2.2454E-4	1.3089E-6	2.8442E-9	2.32
10	7.9031E-2	-9.2430E-4	2.9541E-6		2.54
11	9.2066E-4	-1.1359E-5	3.7823E-8		3.03
12	7.4776E-2	-1.4567E-3	1.0057E-5	2.4116E-8	1.39
13	3.2951E-1	4.2181E-3	1.4222E-5		1.00

TABLE 5.5

POLYNOMIAL COEFFICIENTS FOR n

SOLN	POLYNOMIAL COEFFICIENTS			MEAN ERROR
	k_0	k_1	k_2	%
1	0.902	-4.5580E-4		0.46
2	0.959	-2.1490E-3		0.62
3	1.132	-1.0560E-2	3.251E-5	1.12
4	1.037	-6.2270E-4		1.76
5	0.950	-4.9660E-4		0.52
6	0.929	-1.4170E-4		0.89
7	0.937	2.0590E-4		0.83
8	0.393	3.3130E-4		0.53
9	0.379	-2.6650E-4		1.97
10	0.799	-6.6660E-4		1.77
11	0.948	3.2450E-4		0.50
12	0.970	-6.6000E-5		0.78
13	0.917	-2.2230E-4		2.33

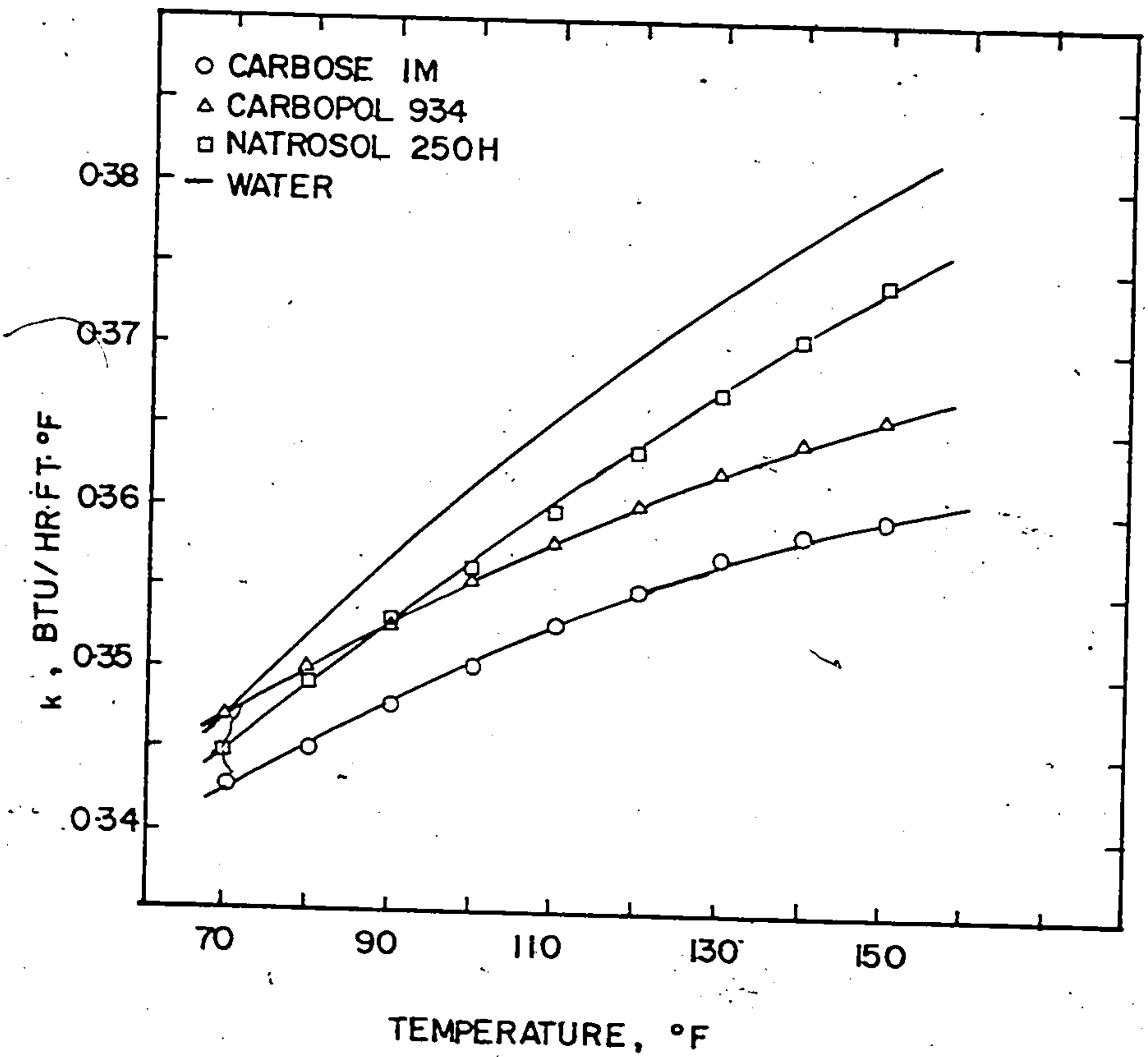


FIG. 5·9 THERMAL CONDUCTIVITY OF POLYMER SOLNS.

Appendix IV) at different temperatures are listed in Table A4.4 to A4.16. The power-law model was plotted in Fig. A4.6 to A4.11 for several solutions to see the adequacy of the model.

Figures 5.10 to 5.13 show K and n to be functions of temperature. The consistency index, which resembles the viscosity of Newtonian fluids, is usually a strong function of temperature while the flow behaviour index is mostly a mild function of temperature. All consistency indices decreased as temperature increased, except the 1% and 1.5% Carbopol 934. This peculiar behaviour of Carbopol 934 was also observed by St. Pierre (81), Reilly (61) and Sharma (78). The parameters K and n of the 13 solutions are tabulated in Table A4.17 to A4.20 in Appendix IV. They are also expressed as a function of temperature in polynomial form in order to facilitate later calculations. The polynomial coefficients for K and n are listed in Tables 5.4 and 5.5.

B. Heat Transfer Results

The heat transfer data were taken over a total heat input range of 4 to 1900 Btu/hr. Before any regression analysis was performed, it was important to insure that free convection around the sphere really did exist at all heat fluxes studied. The method used by Tien and Tsuei (84) to check the onset of free convection was to plot the temperature difference of the heating surface and the fluid against the heat input to the sphere as shown in Figures 5.14 and 5.15. Since there is no discontinuity in both figures, free convection did exist even at the lowest heat flux and remained laminar at the highest heat flux. This would be true for all the solutions studies since Figures 5.14 and 5.15 are plots for the least and most viscous test solutions.

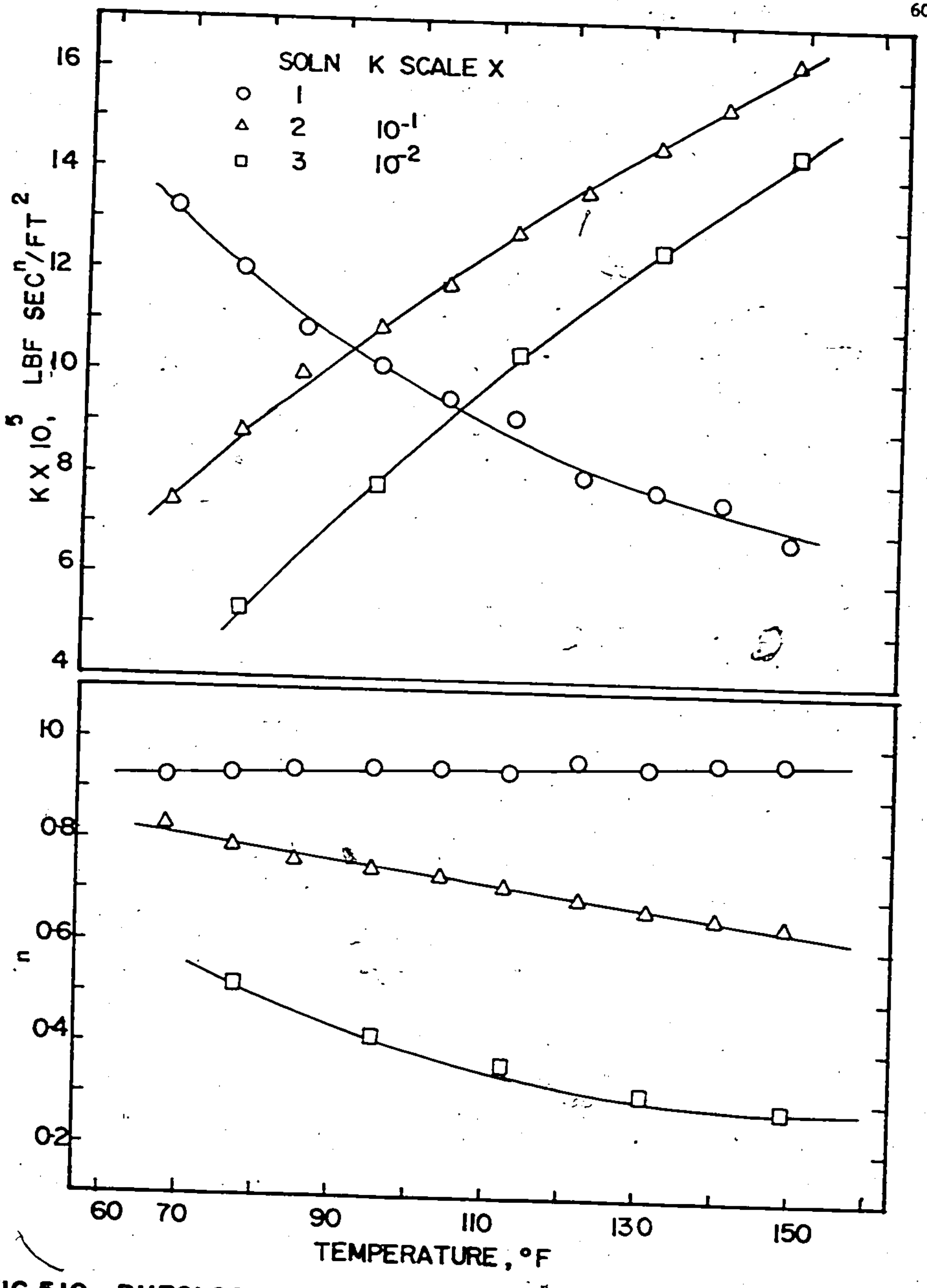


FIG. 5-10 RHEOLOGICAL PROPERTIES FOR CARBOPOL 934 SOLNS.

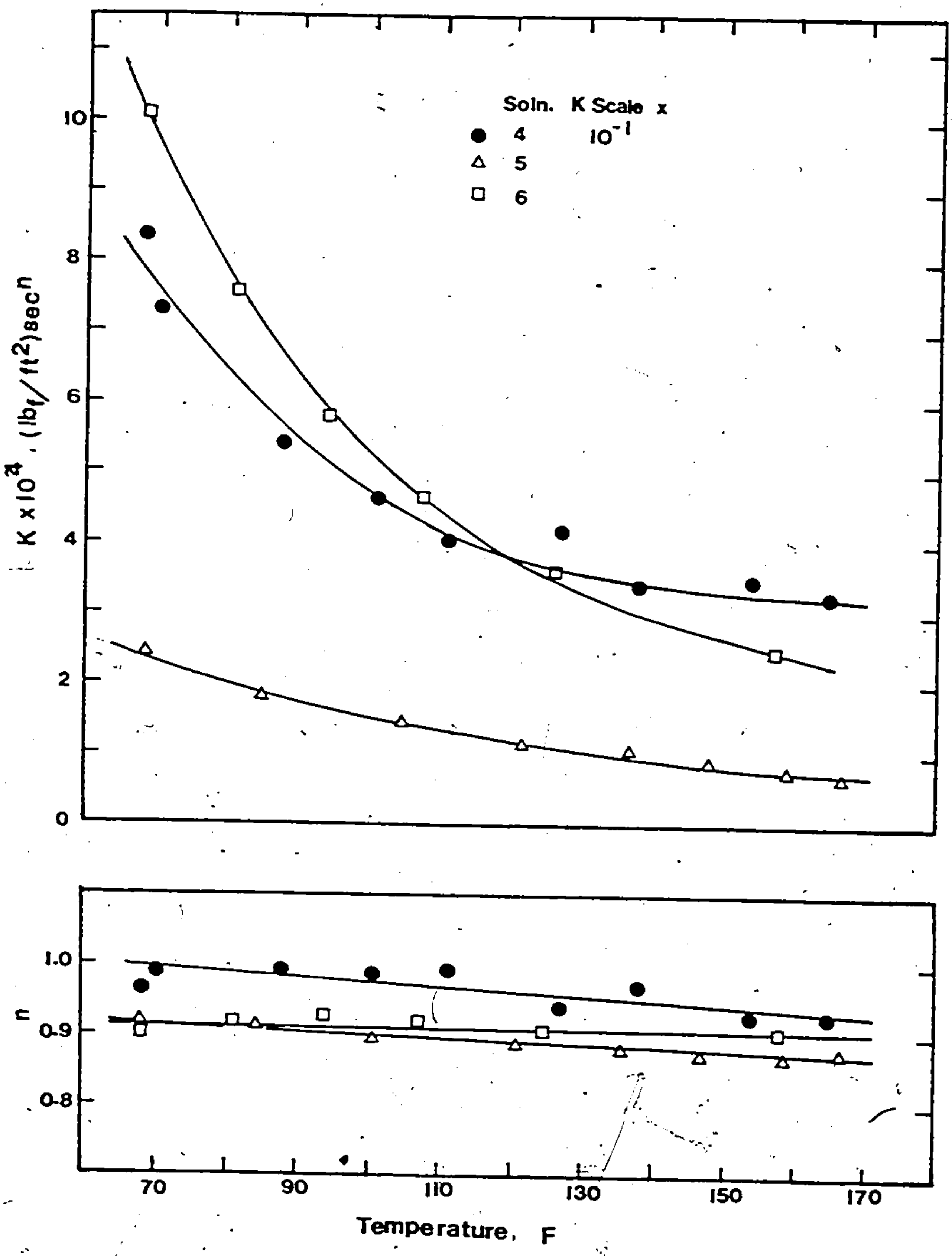


FIG. 5-11 RHEOLOGICAL PROPERTIES FOR CARBOSE IM SOLNS.

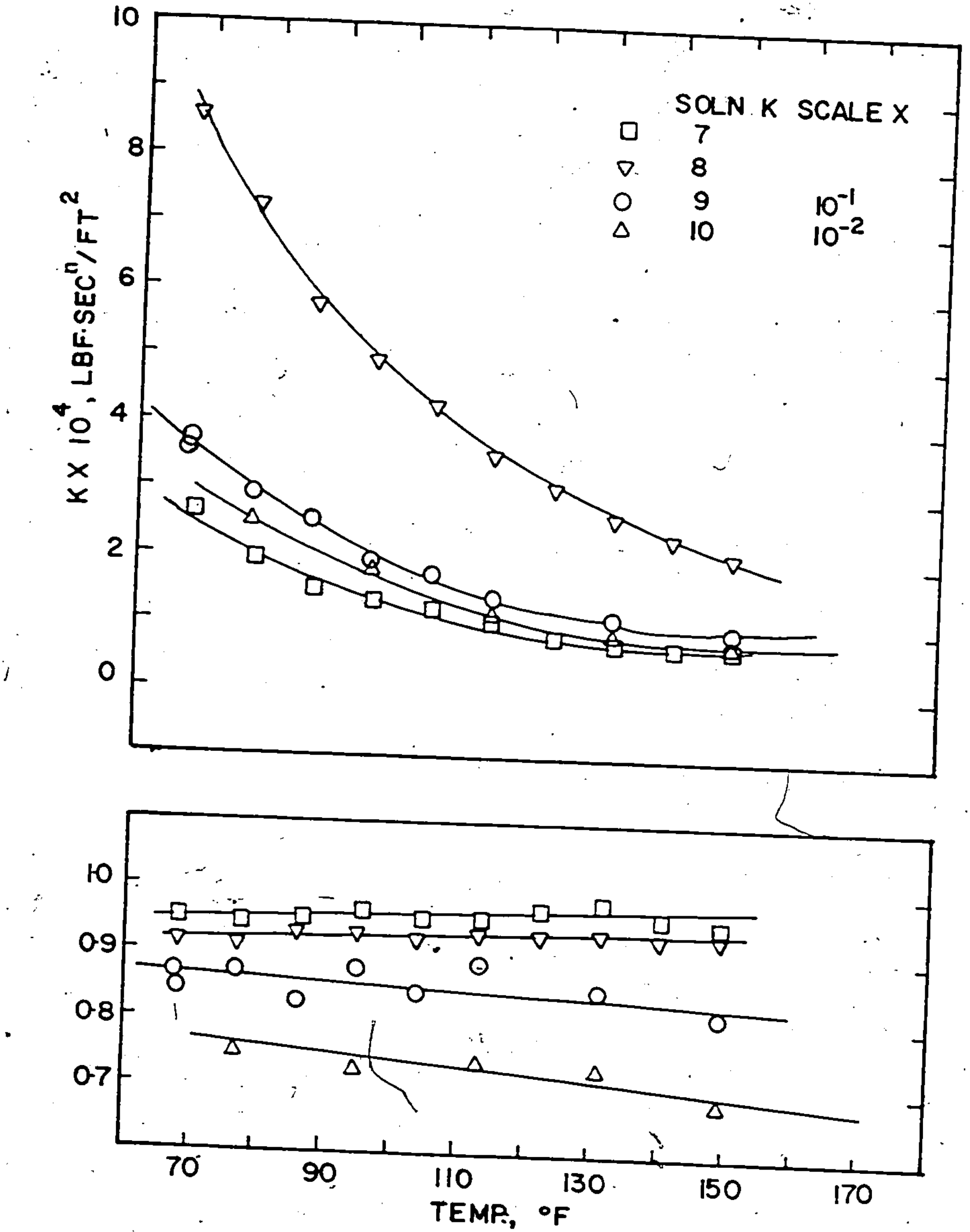


FIG.5-12 RHEOLOGICAL PROPERTIES FOR CARBOSE IM SOLNS.

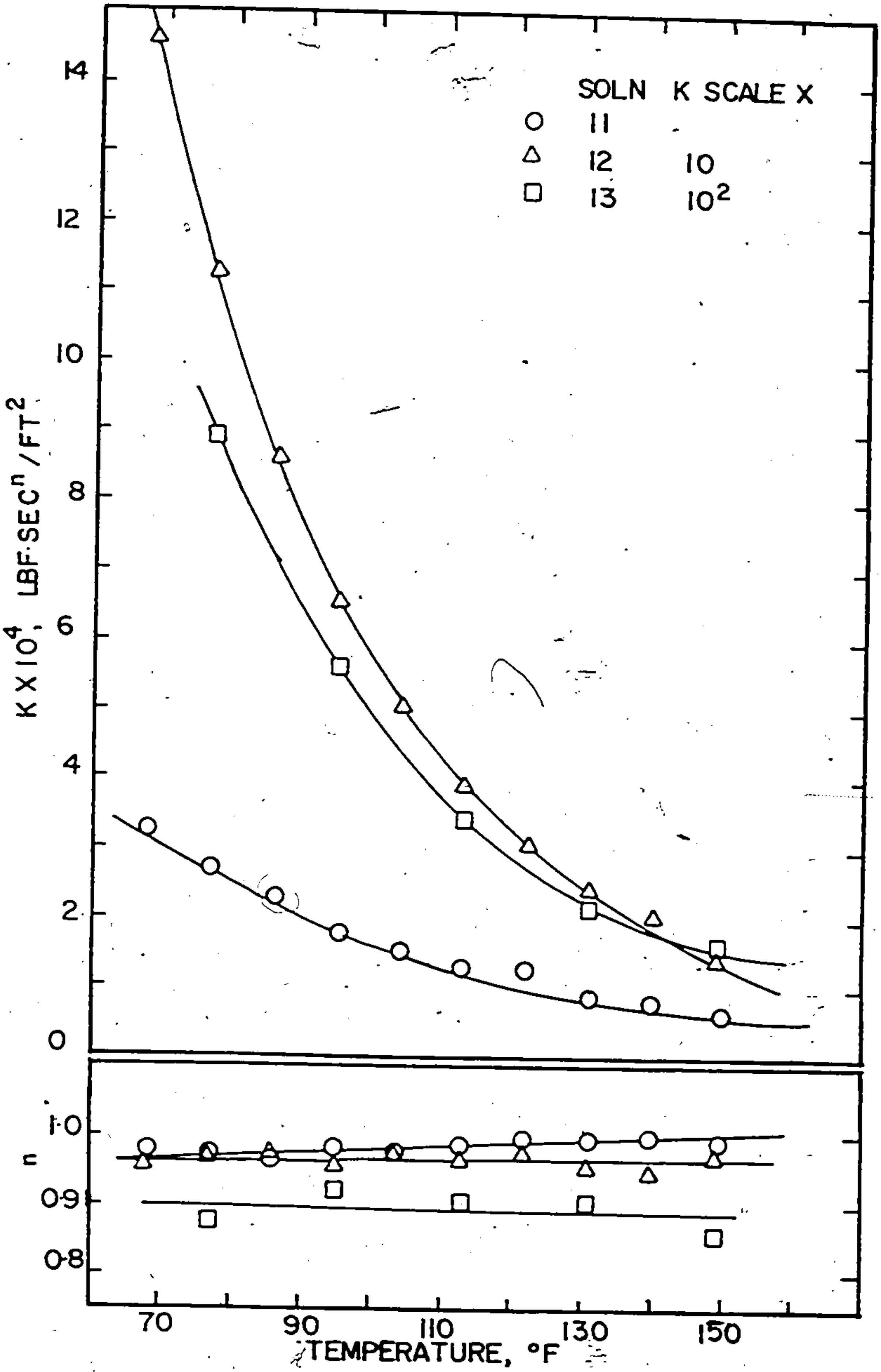


FIG. 5-13 RHEOLOGICAL PROPERTIES FOR NATROSOL 250H

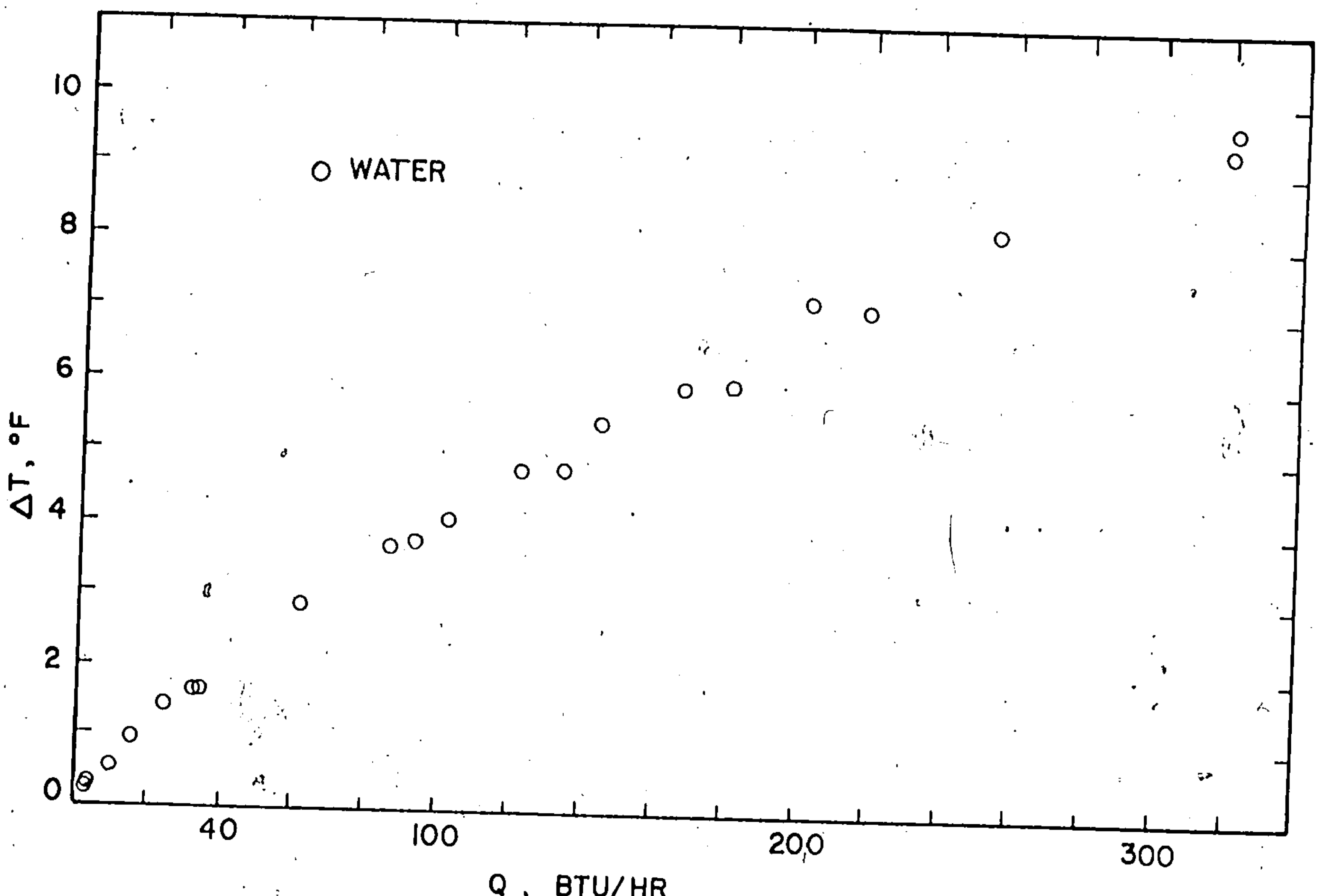


FIG. 5·14 ΔT VS Q — WATER

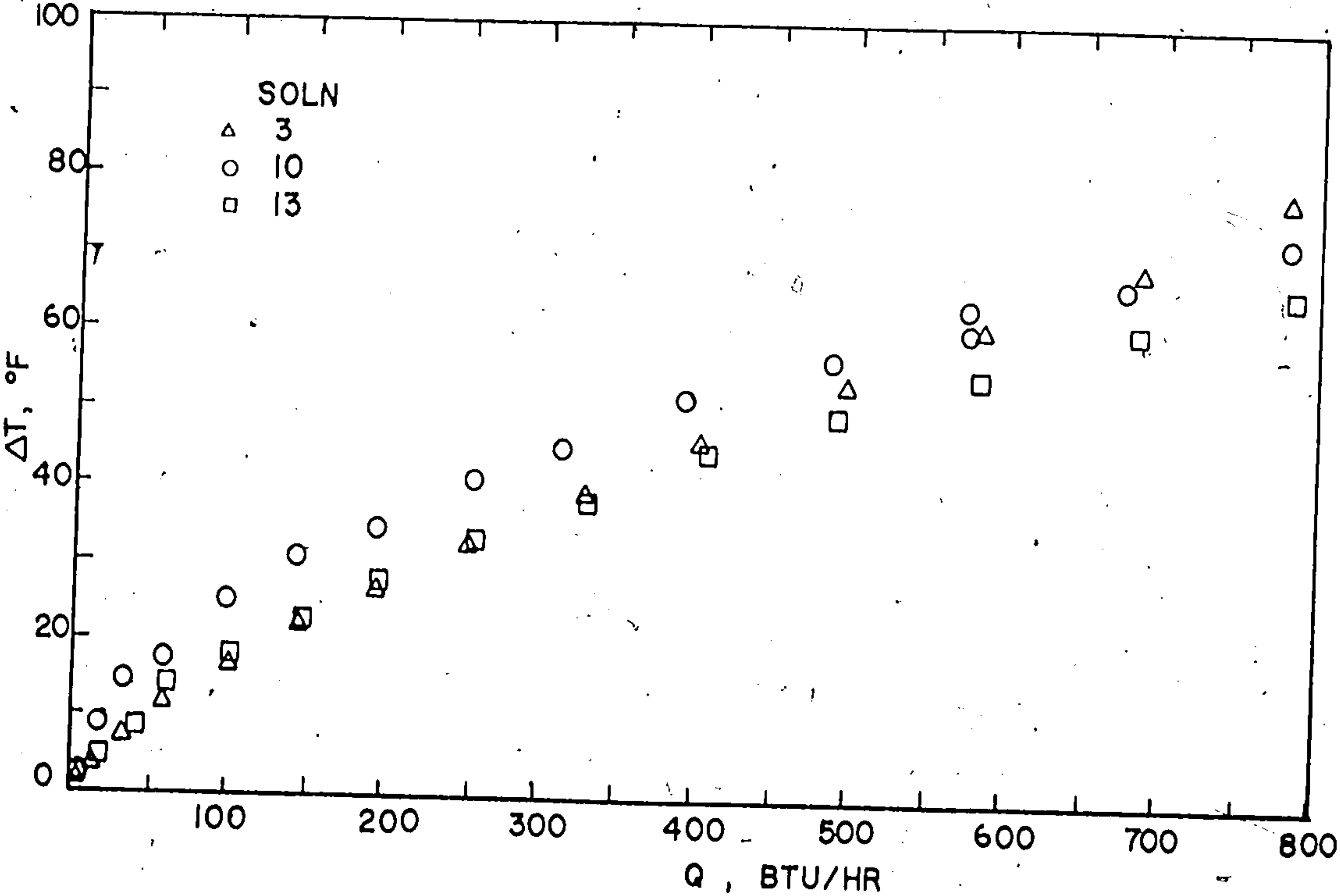


FIG. 5-15 ΔT VS Q — SOLUTIONS 3,10, AND 13

The physical properties of the test solutions were evaluated at the arithmetic mean temperature of the heating surface and the bulk temperature of the fluid. The bulk temperature of the fluid at low heat input to the sphere was the uniform temperature of the surrounding fluid beyond the boundary layer. For high heat input runs, the temperature of the surrounding fluid beyond the boundary layer depended on the depth of the fluid. The temperature of the fluid close to the free surface is higher than that near the bottom of the tank. For a given depth, the temperature of the fluid which was far away enough from the sphere was constant. The bulk temperature was determined from the average of the twenty-one consecutive measurements taken at one inch intervals and about nine inches away from, and parallel to, the north-south axis of the sphere. The locations at which these measurements were taken were found not to be affected by the flow created by the heated sphere. Thus the bulk temperature determined this way for each run was unique.

The heat transfer data for both Newtonian and non-Newtonian fluids are listed in Table A5.1 to A5.27 in Appendix V.

a. Newtonian Fluid

A regression analysis was made on the water data using Equation (3.15):

$$\overline{N_{Nu}} = C'(N_{Gr}\ N_{Pr})^{1/4} \tag{3.15}$$

where
$$N_{Gr} = \frac{g\ \beta\ L^3\ \rho^2\ \Delta T}{\mu^2} \tag{3.13}$$

$$N_{Pr} = \frac{C_p\ \mu}{k} \tag{3.14}$$

$$\overline{N_{Nu}} = \frac{\bar{h}\,L}{k} \qquad (5.5)$$

The overall heat transfer coefficient $\bar{h}$ used in equation (5.5) was evaluated from equation (5.6)

$$Q = \bar{h}\,Ar\,(T_s - T_h) \qquad (5.6)$$

where Ar is the total surface area of the sphere minus the area covered by the 1.5 inch diameter silicone gasket.

The value of the parameter C was evaluated with L equal to the radius and diameter of the sphere. With L = radius, the data is plotted in Figure 5.16 and C equalled 0.494, which agreed well with Acrivos' theoretical value (1). The mean error (4.5%) as well as other statistically analysed data for this correlation is also given in Table 5.6. With L = diameter, C is 0.588. This value is 0.8% lower than the first approximation prediction and 5% higher than the second approximation by Merk and Prins (49). Direct comparisons to other experimental results which have approximately the same range of $N_{Gr}N_{pr}$ (10^5 to 10^8), is given in Table 5.7. It is seen that the results agree well with both theoretical predictions and the C value reported by other investigators (8, 9).

Table 5.8 which includes not only heat transfer data but also mixed heat and mass as well as pure mass transfer data is presented for qualitative comparison purposes.

b. Non-Newtonian Fluids

The heat transfer results for polymer solutions were analysed in terms of Acrivos' equation and the empirical equation used by Reilly (61) and Sharma (78).

1. Acrivos' Method

Equation (3.40) whose derivation after Acrivos method in Chapter III is :

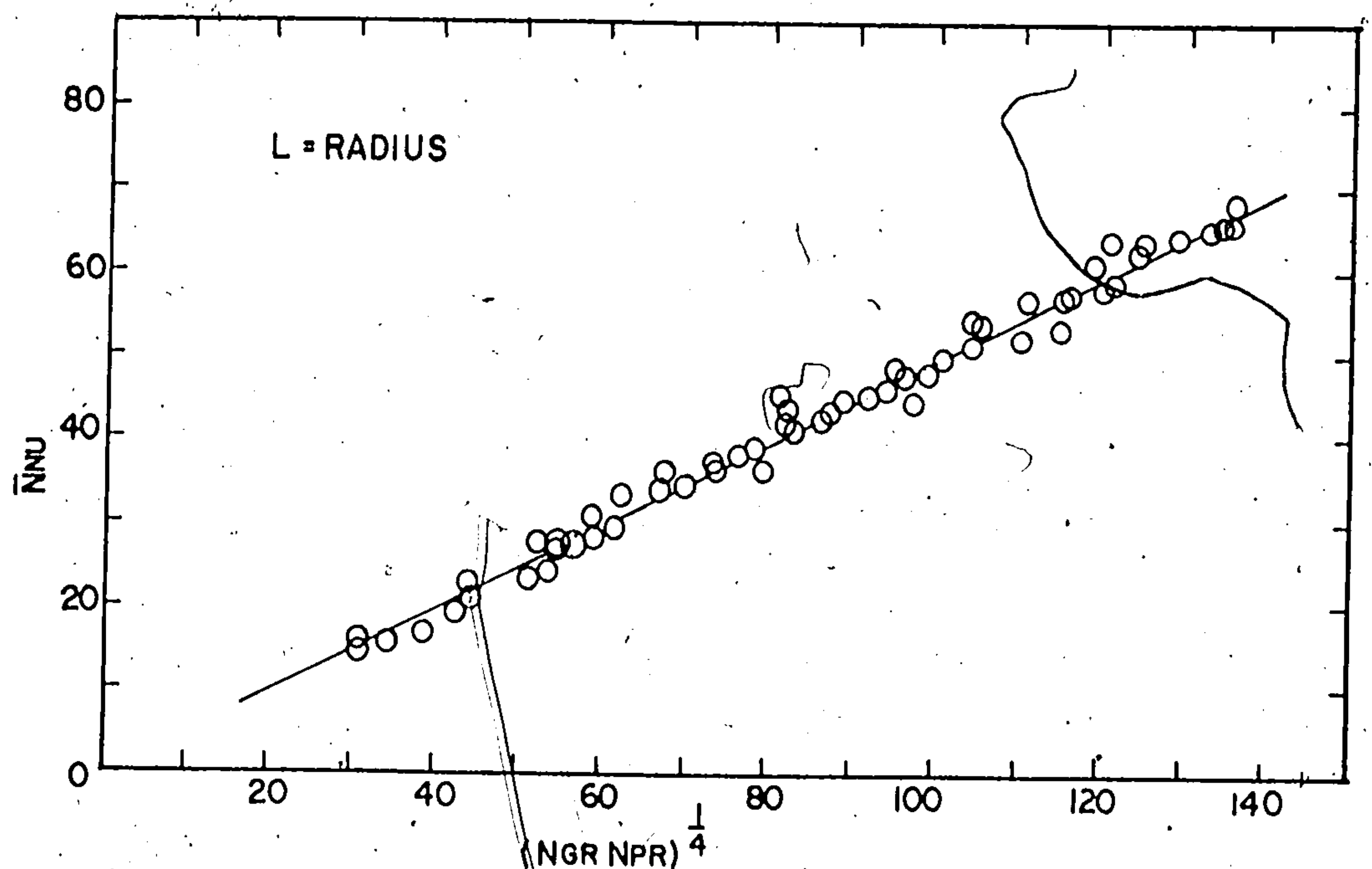


FIG. 5.16 FREE CONVECTION HEAT TRANSFER — WATER

TABLE 5.6

EXPERIMENTAL VALUES OF C

L=RADIUS

SOLN	C	σ OF C **	PROBABLE DEV.	MEAN ERROR, %	RANGE OF N_{Ra} *	RANGE OF N_{Nu}	AVE. n	CORR. NO.
1	0.544	0.016	[illegible]	4.7	1.33E5-2.03E7	10.0-50.4	0.857	1
2	0.608	0.015	0.617	2.[illegible]	1.36E5-1.70E6	15.0-29.6	0.736	2
3	0.434	0.024	0.600	5.8	4.67E3-2.04E5	3.3-12.2	0.411	3
4	0.530	0.009	1.009	2.4	7.20E5-8.15E7	17.7-55.5	0.973	4
5	0.571	0.012	1.260	3.7	1.54E5-2.50E7	10.7-48.9	0.900	5
6	0.565	0.012	1.102	5.3	2.73E4-1.33E8	7.5-38.3	0.915	6
7	0.578	0.011	0.983	3.1	9.23E4-4.31E7	11.2-48.2	0.957	7
8	0.563	0.006	0.477	2.2	3.34E4-1.83E7	3.3-39.4	0.927	8
9	0.527	0.014	0.690	4.4	3.36E4-4.08E5	6.6-27.9	0.852	9
10	0.370	0.029	0.902	17.1	3.33E3-7.13E5	2.5-12.1	0.728	10
11	0.597	0.012	1.074	2.9	1.75E5-5.11E7	15.5-48.4	0.979	11
12	0.613	0.008	0.328	2.4	4.30E5-3.07E6	6.9-26.7	0.963	12
13	0.621	0.017	0.421	4.2	7.09E2-4.21E5	3.0-14.8	0.894	13
ALL	0.558	0.005	1.907	10.7	7.09E2-1.33E8	2.5-55.5		14
ALL EXCEPT 3 & 10	0.561	0.004	1.602	5.9	7.09E2-1.33E8	2.5-55.5		15
WATER	0.494	0.013	3.070	4.5	8.81E5-3.41E8	14.7-57.0		16

* $N_{Ra} = N_{Gr}N_{Pr}$

** σ : standard deviation

TABLE 5.7

COMPARISON OF FREE CONVECTION-----WATER

INVESTIGATOR	C	L	MEAN ERROR %*
ACRIVOS (1)	0.49	RADIUS	0
PRESENT STUDY	0.494	RADIUS	---
MERK & PRINS (49)	0.592**	DIAMETER	0.7
MERK & PRINS (49)	0.558***	DIAMETER	-5.1
PRESENT STUDY	0.588	DIAMETER	---
AGRAWAL & ADELMAN (6)	0.64	DIAMETER	-18.6
AMATO & TIEN (8)	0.50	DIAMETER	-15.9
BOBERG & STARRETTE (9)	0.51	DIAMETER	-13.3

* Mean Error $= \dfrac{C - C_{author}}{C_{author}} \times 100\%$

** By First Approximation for $N_{Pr} \rightarrow \infty$.

*** By Second Approximation for $N_{Pr} \rightarrow \infty$.

Table 5.8 COMPARISON OF EXPERIMENTAL INVESTIGATION OF FREE CONVECTION HEAT AND MASS TRANSFER IN NEWTONIAN FLUIDS (L = DIAMETER)

	Investigator	Equation	Condition
1	Yuge (21)	$\overline{N}_{Nu} = 2 + 0.428\ N_{Ra}^{1/4}$	Heat transfer - air
2	Kyte, Madden & Piret (22)	$\overline{N}_{Nu} = 2 + 0.399\ N_{Ra}^{1/4}$	Heat transfer - air
3	Amato & Tien (8)	$\overline{N}_{Nu} = 2 + 0.5\ N_{Ra}^{1/4}$	Heat transfer - water
4	Jacob & Linke (23)	$\overline{N}_{Nu} = 0.555\ N_{Ra}^{1/4}$	Heat transfer - various shape
5	Boberg & Starrette (24)	$\overline{N}_{Nu} = 0.51\ N_{Ra}^{1/4}$	Heat transfer - transient method (water)
6	Agrawal & Adelman (18)	$\overline{N}_{Nu} = 0.64\ N_{Ra}^{1/4}$	Heat transfer - water
7	Liew & Adelman	$\overline{N}_{Nu} = 0.588\ N_{Ra}^{1/4}$	Heat transfer - water
8	Ranz & Marshall (25)	$\overline{N}_{Nu} = 2 + 0.6\ N_{Pr}^{1/3}\ N_{Ra}^{1/4}$	Evaporation - drops
9	Schenk & Schenkels (26)	$\overline{N}_{Nu} = 0.56\ N_{Ra}^{1/4}$	Melting ice sphere
10	Vanier (27)	$\overline{N}_{Nu} = 2 + 0.52\ N_{Ra}^{1/4}$	Melting ice sphere
11	Kranse & Schenk (28)	$\overline{N}_{Nu} = 2 + 0.59\ N_{Ra}^{1/4}$	Melting-benzene
12	Van der Burgh (29)	$\overline{N}_{Nu} = 0.525\ N_{Ra}^{1/4}$	Melting-benzene

CONTINUED - NEXT PAGE

	Investigator	Equation	Condition
13	Schenkels & Schenk (30)	$\overline{N}_{Nu} = 2 + 0.59\ N_{Ra}^{1/4}$	Melting-organic spheres
14	Garner & Keey (31)	$\overline{N}_{Sc} = 23 + 0.585\ N_{Ra}^{1/4}$	Mass transfer
15	Garner & Hoffman (32)	$\overline{N}_{Nu} = 5.4 + 0.44\ N_{Ra}^{1/4}$	Mass transfer
16	Schutz (33)	$\overline{N}_{Sc} = 2 + 0.59\ N_{Ra}^{1/4}$	Mass transfer

$$\overline{N_{Nu}} = C\, N_{Gr}^{\frac{1}{2(n+1)}}\, N_{Pr}^{\frac{n}{3n+1}} \tag{3.40}$$

$$N_{Gr} = \frac{\rho^2 L^{n+2} [g_c\, \beta (T_s - T_\infty)]^{2-n}}{K^2} \tag{3.21}$$

$$N_{Pr} = \frac{\rho C_p}{k} \left(\frac{K}{\rho}\right)^{\frac{2}{1+n}} (L)^{\frac{1-n}{1+n}} [L\beta\, g_c (T_s - T_\infty)]^{\frac{3(n+1)}{2(n+1)}} \tag{3.22}$$

$\overline{N_{Nu}}$ is defined by equation (5.5).

It has been pointed out by the author (3) that although the parameter C in equation (3.40) depends on n, the equation could be used for data with varying n. The use of equation (3.40) in the following correlations with n as a variable is somehow modified.

The experimental results are plotted in Figures 5.17 to 5.20 and the C values are listed in Table 5.6. A single correlation of the 13 solutions yielded an average of C of 0.558 at a mean error of 10.7%. The correlation of eleven solutions (without 1.5% Carbopol 934 and 2.5% Carbose IM) gave a mean error of 5.9% which was almost as good as the individual correlation. The reason for omitting solution 3 and 10 will be discussed in the next chapter.

Laminar free convection heat transfer in 0.5% and 1% CMC (Carboxymethyl Cellulose) solutions has been reported by Amato (8). The average value of C (0.575 and 0.564) for 0.5% and 1%, Carbose 1M is about 13% and 17% higher than that of Amato's. There is no available data to compare with other solutions.

Amato also claimed that equation (3.40) failed to correlate data obtained from CMC-7H and Polyox WSR-FRA solutions for $N_{Gr}^{\frac{1}{2(n+1)}} N_{Pr}^{\frac{n}{3n+1}} < 10$. The data at this region is correlated by equation (5.7)

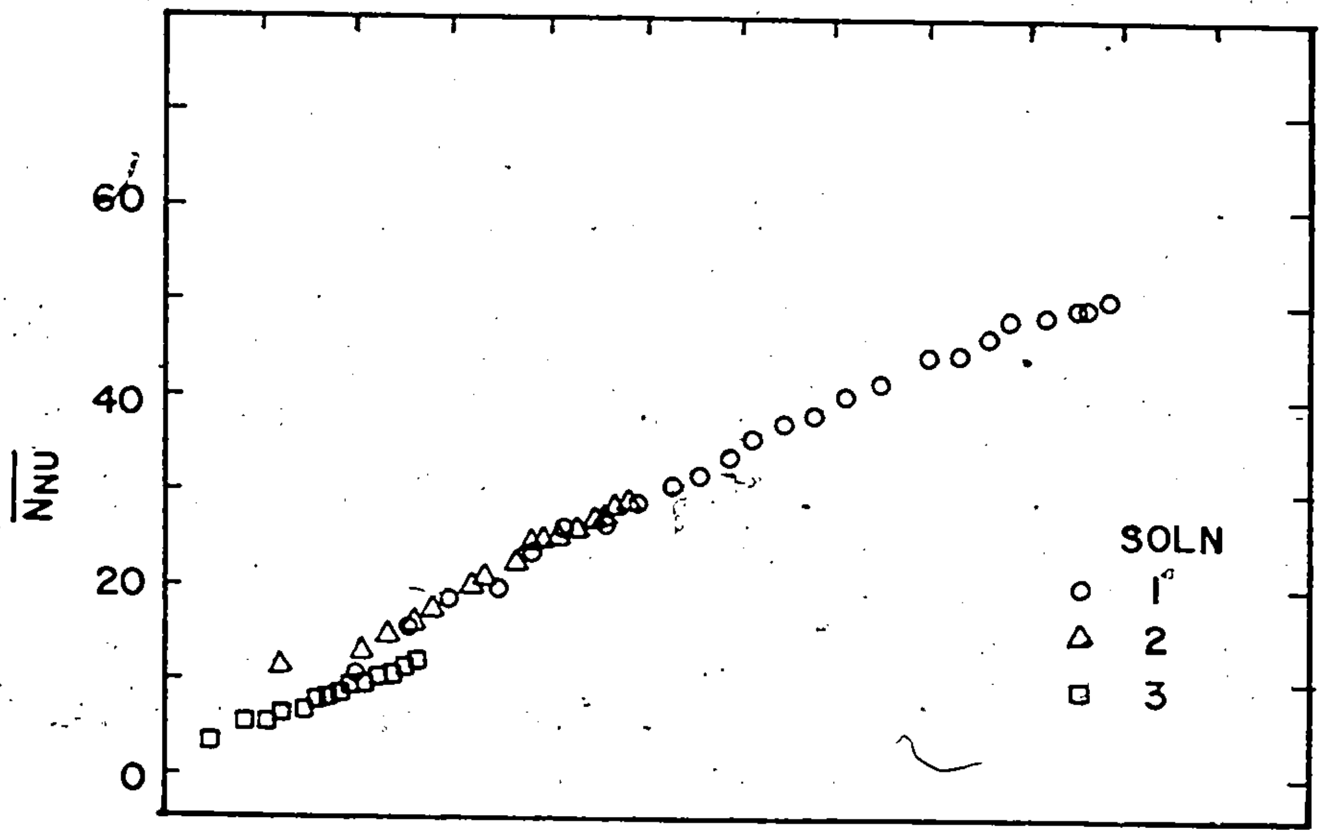


FIG.5·17 FREE CONVECTION HEAT TRANSFER — CARBOPOL 934

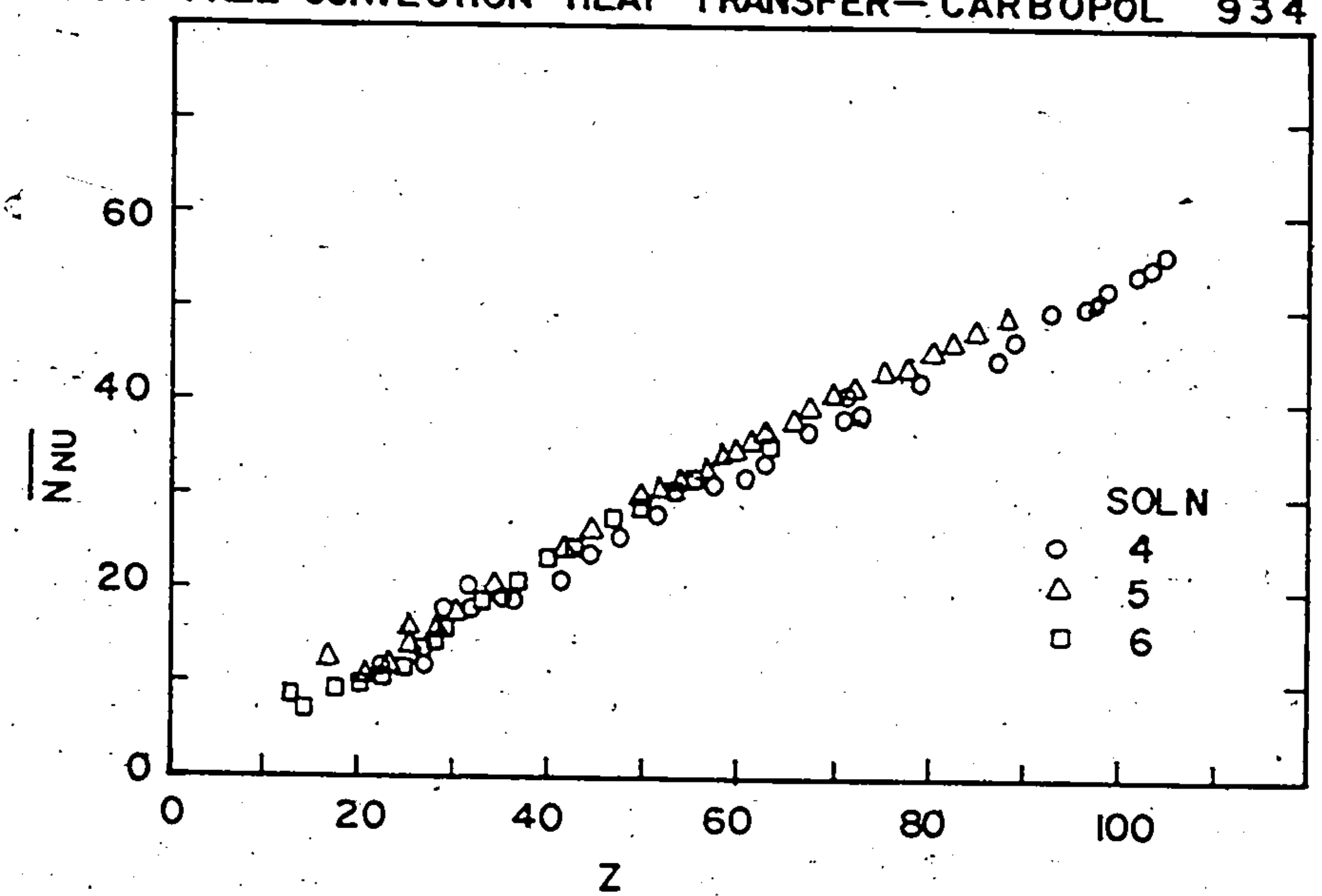


FIG.5·18 FREE CONVECTION HEAT TRANSFER — CARBOSE IM

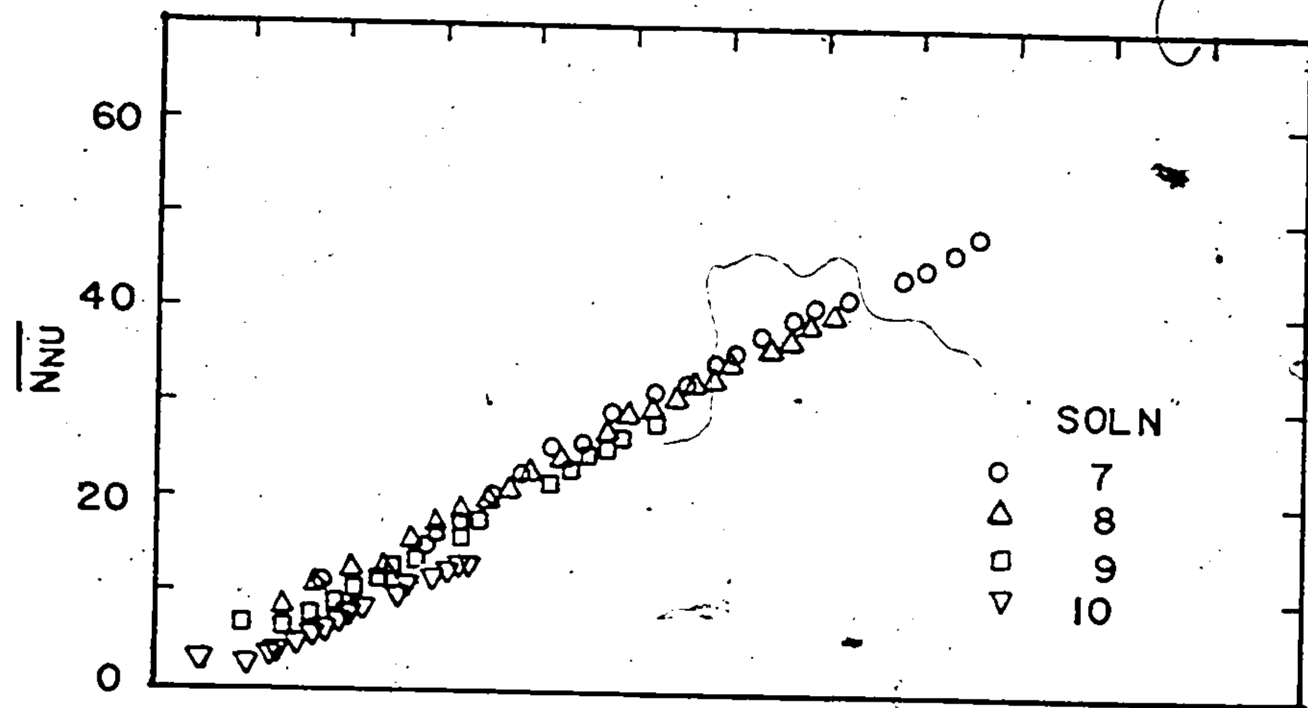


FIG.5-19 FREE CONVECTION HEAT TRANSFER — CARBOSE 1M

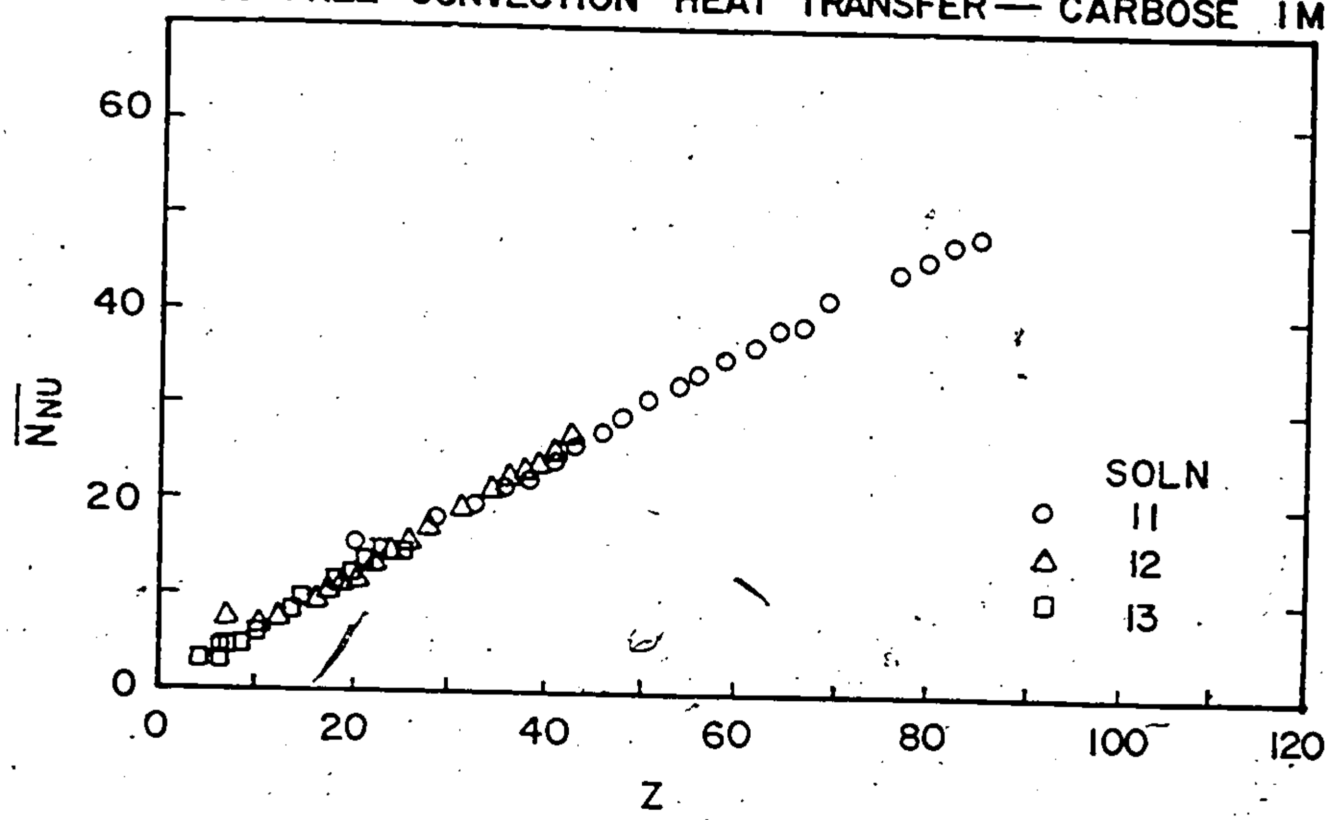


FIG. 5-20 FREE CONVECTION HEAT TRANSFER — NATROSOL 250 H

$$\overline{N_{Nu}} = C_3 \left[N_{Gr}^{\frac{1}{2(n+1)}} N_{Pr}^{\frac{n}{3n+1}} \right]^{C_4} \qquad (5.7)$$

Where C_3 and C_4 are constants. Unfortunately, the data obtained from the present study do not confirm this point but show that eqn. (3.40) could correlate data with $N_{Gr}^{\frac{1}{2(n+1)}} N_{Pr}'^{\frac{n}{n+1}} \doteq Z$ as low as 5 (Fig.5.21).

2. Empirical Method

The experimental results were also correlated in the form of eqn. (3.50)

$$\overline{N_{Nu}} = C_1 \, (N_{Gr} \, N_{Pr}')^{C_2} \qquad (3.50)$$

where as $\overline{N_{Nu}}$ and N_{Gr} are defined by eqns. (5.5) and (3.21). The Prandtl number is:

$$N_{Pr}' = \frac{C_p c}{k} \left[\frac{K}{\rho}\right]^{\frac{1}{2-n}} [L]^{\frac{2(n-1)}{n-2}} \qquad (3.51)$$

Fig. 5.22 is a plot of equation (3.50) for all the thirteen solutions. The values of C_1 and C_2 are listed in Table 5.9. The single correlation for all the thirteen solutions gives a mean error of 17% while the correlation without solution 3 and 10 gives a mean error of 6.2%. It is very interesting to observe that the value of C_2 is approximately equal to a quarter, which is the widely accepted value for Newtonian fluids.

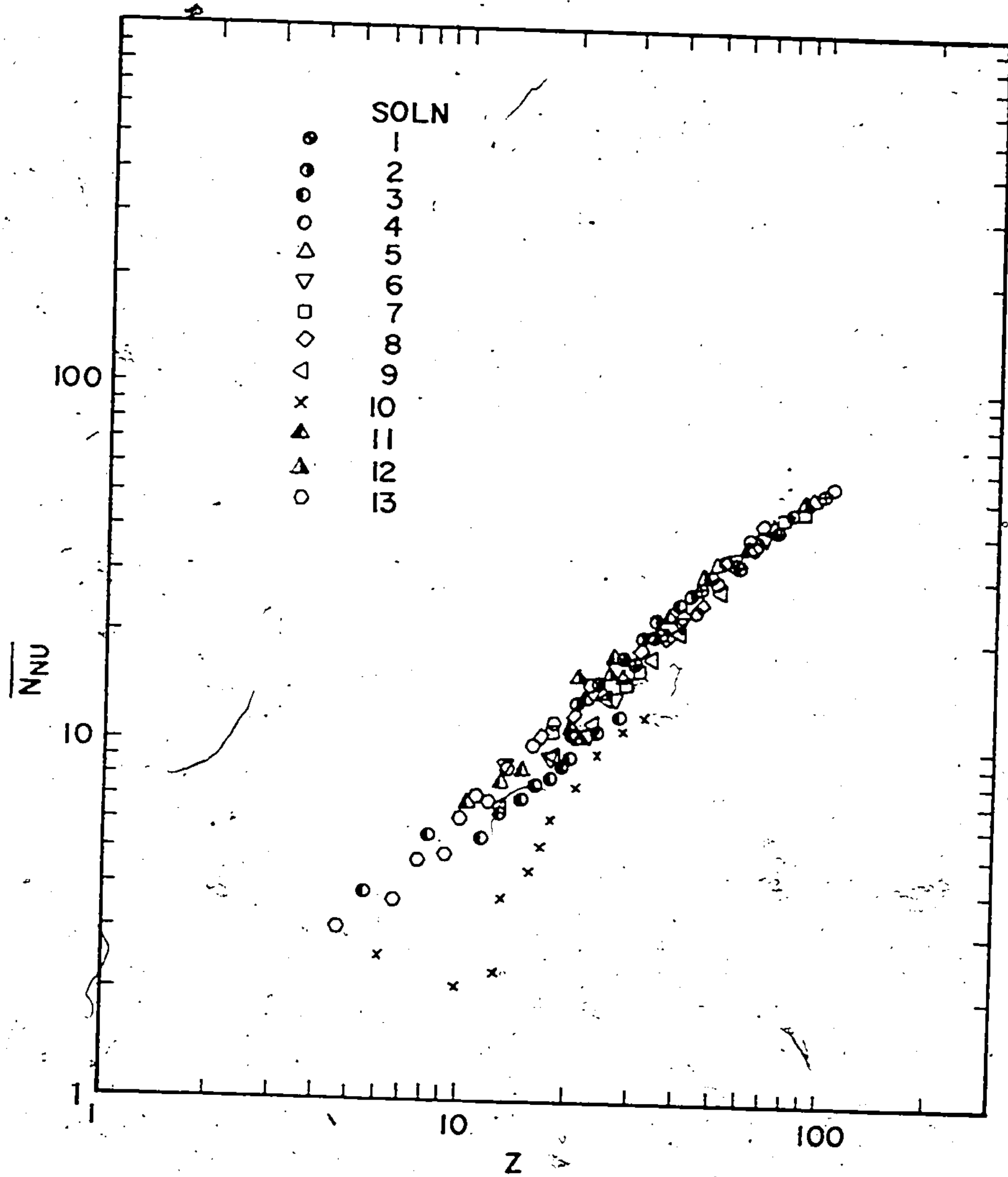


FIG.5·21 FREE CONVECTION HEAT TRANSFER — POLYMER SOLUTIONS

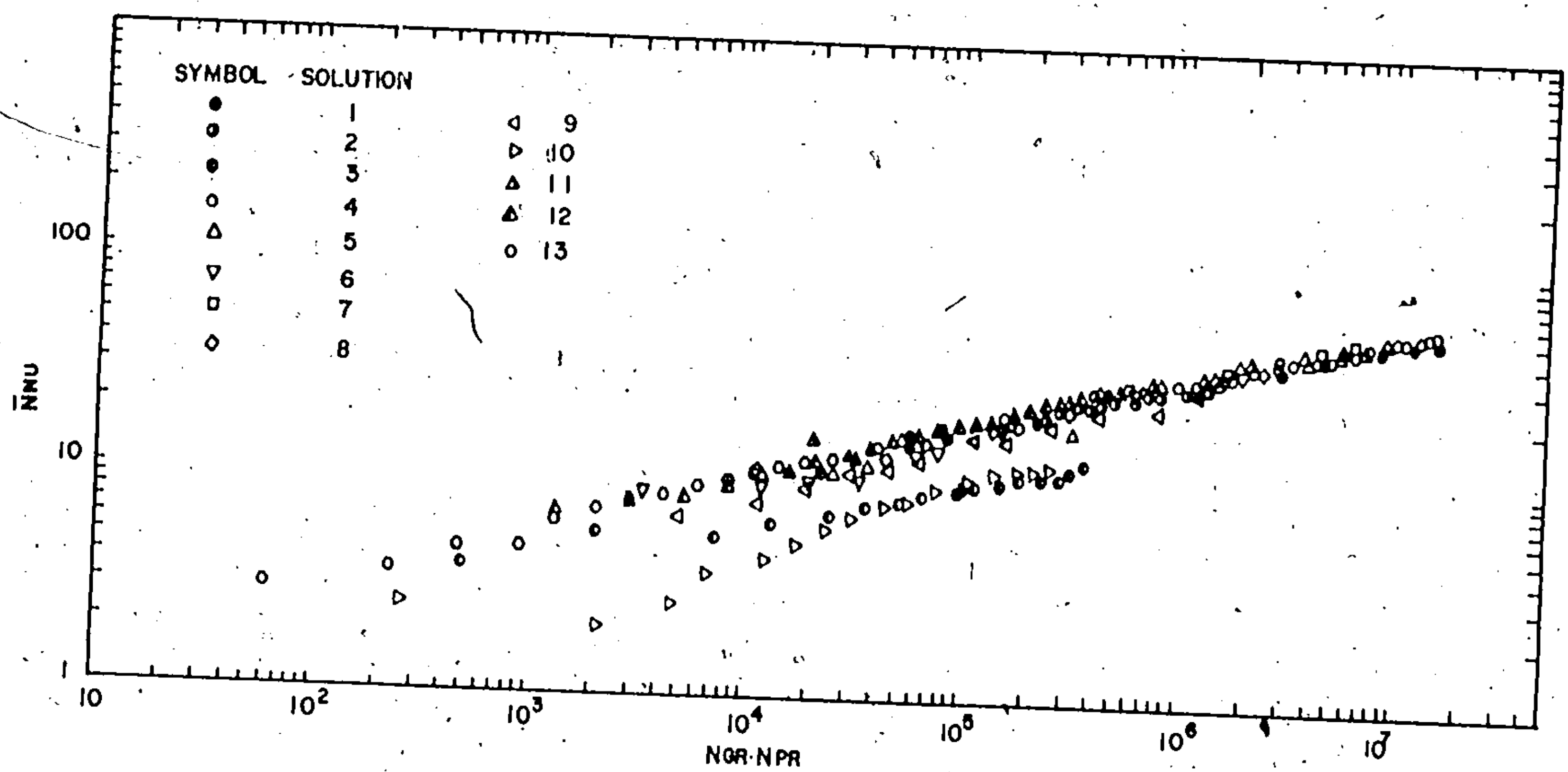


FIG 5·22 FREE CONVECTION HEAT TRANSFER — BY EQN·(3·50)

Table 5.9 REGRESSION ANALYSIS BY EMPIRICAL METHOD

Solution	C_1	σ of C_1 **	C_2	σ of C_2	Probable Deviation	Mean Error %	Range of N_{Ra}' *	Range of N_{Nu}	Correlation No.
1	0.794	0.084	0.223	0.005	1.237	3.1	$2.3 \times 10^5 - 1.6 \times 10^8$	10.9 - 50.4	1
2	0.844	0.080	0.218	0.005	0.579	1.8	$3.4 \times 10^5 - 1.4 \times 10^7$	13.0 - 29.6	2
3	0.889	0.092	0.170	0.007	0.344	3.1	$4.9 \times 10^3 - 3.6 \times 10^6$	3.8 - 12.2	3
4	0.746	0.070	0.229	0.004	0.935	2.3	$7.9 \times 10^5 - 1.5 \times 10^8$	17.7 - 55.5	4
5	0.462	0.107	0.258	0.007	1.484	4.2	$2.5 \times 10^5 - 8.9 \times 10^7$	10.7 - 48.9	5
6	0.467	0.112	0.257	0.008	1.047	5.0	$3.5 \times 10^4 - 2.1 \times 10^7$	7.6 - 38.3	6
7	0.590	0.086	0.246	0.005	0.920	2.8	$1.2 \times 10^5 - 6.3 \times 10^7$	11.2 - 48.2	7
8	0.614	0.053	0.241	0.004	0.509	2.1	$4.4 \times 10^4 - 3.2 \times 10^7$	8.3 - 39.4	8
9	0.343	0.062	0.271	0.004	0.468	1.7	$5.1 \times 10^4 - 1.2 \times 10^7$	6.6 - 27.9	9
10	0.146	0.327	0.302	0.026	0.669	12.2	$2.6 \times 10^3 - 2.6 \times 10^6$	2.5 - 12.1	10
11	0.827	0.098	0.229	0.006	0.841	2.6	$2.0 \times 10^5 - 5.6 \times 10^7$	15.5 - 48.4	11
12	0.578	0.054	0.252	0.004	0.295	1.9	$1.3 \times 10^4 - 3.9 \times 10^6$	6.9 - 26.7	12
13	0.548	0.081	0.254	0.008	0.470	4.2	$6.0 \times 10^2 - 6.1 \times 10^5$	3.0 - 14.8	13
1-13	0.390	0.081	0.266	0.005	3.011	15.7	$6.0 \times 10^2 - 1.6 \times 10^8$	2.5 - 55.5	14
1-13 except 3 6 10	0.611	0.034	0.241	0.002	1.886	6.2	$6.0 \times 10^2 - 1.6 \times 10^8$	3.0 - 55.5	15

* $N_{Ra}' = N_{Gr} N_{Pr}'$: ** σ : standard deviation

VI. DISCUSSION AND CONCLUSIONS

A. Discussion

a. Physical Properties Measurement

The thermal expansion coefficient obtained was different from that of Acrivos', which is an approximation by equation (3.19). The evaluation of β by eqn. (4.1) is somehow dependent on the sensitive term $(\frac{\partial\rho}{\partial T})_p$ of the polynomial expression for density. The β of water was evaluated by both second and third order polynomials and the difference was found to be 2%.

The thermal conductivity apparatus has the following two disadvantages: (1). The equipment takes six hours to reach steady state. (2). The heat flux q to the heating block has to be as constant as possible for all the runs. The difference in heat flux for all runs in the present study was within 1%.

The rheological properties of the thirteen pseudoplastic solutions were determined with the range of shearing rate from about 0.1 to 70 sec^{-1}. Although Figs. A4.6 to A4.11 show that the power-law can represent the relationship of shearing stress and rate fairly well in most solutions, Figures A4.7 and A4.9 clearly show that the accuracy of the power-law model decreases as the concentration of the solution increases. The n values in these two figures could be substantially different depending on the shearing rate. From equation (3.37) it is obvious that C would be strongly dependent on n. Therefore an n value obtained from an inappropriate shearing rate would mean an inappropriate C.

b. Heat Transfer Measurement

1. Newtonian Fluids

The experimental value of C' for water with changing physical properties is in good agreement with both Acrivos' and Merk and Prins' prediction, although both theoretical analyses are based on constant physical properties. This would seem to indicate that the change of physical properties of water with temperature is mild enough not to significantly affect the C' value.

2. Non-Newtonian Fluids

Direct comparison of the regression analysis of the results obtained using Acrivos' method and the empirical method is available in Tables 5.6 and 5.9. Although the empirical equation (eqn. 3.50) gives a slightly better individual correlation for most solutions, the reverse is true for the overall correlations 14 and 15. The mean errors for correlations 14 and 15 for equation (3.40) are 10.7 and 5.9% while for equation (3.50) they are 15.7 and 6.2%. It is actually not surprising if one examines the two equations closely. Equation (3.50) has one parameter more than equation (3.40). The extra parameter would play an important role in reducing the error in the individual correlation, but in the overall correlation, the fitness of the correlation is more significantly affected by whether an individual set of data agrees with each other. From these comparisons, it seems that equation (3.40) is more suitable for correlating data than equation (3.50). Equation (3.40) is preferable, not only because it can correlate data slightly better, but also, more importantly because of its theoretical basis and fewer parameters. The discussion henceforth will focus on equation (3.40).

Some of the plots from Figure 5.17 to 5.20 are curved slightly downward. The C value, which is the slope of the plot, would therefore be dependent on Z, i.e. C decreases as z increases. If these curvatures are caused by the modified use of the equation, the solutions that show curvatures in Fig. 5.17 to 5.20 should correspond to solutions which have similar functions of n vs. temperature, ie. either n increases with temperature or vice versa. Comparison of Figure 5.10 to 5.13 and Figure 5. 17 to 5.20 shows no evidence to support this argument. No further analysis was done on this problem because the curvatures are negligibly small.

According to Table 3.4, C decreases as n decreases. The experimental value of C for the polymer solutions should therefore be less than 0.494. But Table 5.6 shows a contradicting result. As has been mentioned before, the C values in Table 3.4 were based on constant physical properties, and it is necessary to see if the variation of physical properties might have affected the C value in Table 5.6. The rheological property is the most temperature dependent physical property. Furthermore, the K value of water is very much different from that of polymer solutions. A comparison of the magnitude of change of K in the temperature range should be able to show whether the C value of polymer solutions is affected to a different degree than the C value of water by the changing physical properties. The ratio of K values at 70°F and 150°F for water, solutions 4, 5 and 6 is shown in Table 6.1. If the C value of water with a ratio of 2.4 is not significantly affected by the change of physical properties, there is no reason why solutions 4, 5 and 6 should be, especially for solution 5 whose ratio is only 2.1. This seems to indicate that the deviation of C in Table 5.6 from Table 3.4 is not a result of changing physical properties. It might be a result of other effects such as intermolecular attractions of the polymer solutions.

Table 6.1 Order of Change of K with temperature

Solution	K ratio
Water	2.4
4	3
5	2.1
6	4

In order to see if there is any relationship between the same type of solutions having different concentrations as well as different types of solution, the C values for Carbopol

934, Carbose IM and Natrosol 250H are first plotted against the concentration of the solutions in term of weight percent. Figure 6.1 shows that the value of C are fairly constant for concentration less than 1.25%, whereas at higher concentration C decreases with concentration. From Figure 6.1, C can simply be treated as a function of concentration, but that is obviously not a reasonable treatment because the same weight % of different type of polymer solution does not necessarily create the same degree of non-Newtonian behaviour. It would be more reasonable to correlate the C value into the physical properties since they are also functions of concentration. The plot of C vs. mean density of the solution in Figure 6.2 has the general shape of Figure 6.1 except that solution 10 deviates from the general trend. The average value of n is plotted in Figure 6.3. In all the three plots, a horizontal line can be fitted with tolerable experimental error if solutions 3 and 10 are excluded. The possible effect of K on C is also tested. Values of C from different experimental points among all the polymer solutions having the n value within 0.01 from each other have been plotted. A typical example is shown in Figure 6.4. No evidence of the dependence of C on K is found.

Further examination of Table 6.2 shows the range of K and n for which Acrivos' equation is valid is not well defined. Although the C value of solution 3, which has the highest K and lowest n, deviate a little from that of solution 1 and 2, the deviation is not as much as that of solution 10, whose highest value of K is less than that of solution 13 and lowest value of n is higher than that of solution 2.

The analyses so far fail to explain the deviation of C for solutions 3 and 10. The failure could be due to change in transport mechanism because of the high concentration of the polymer solutions. For Newtonian fluids it has been observed that equation (3.15) fails to correlate

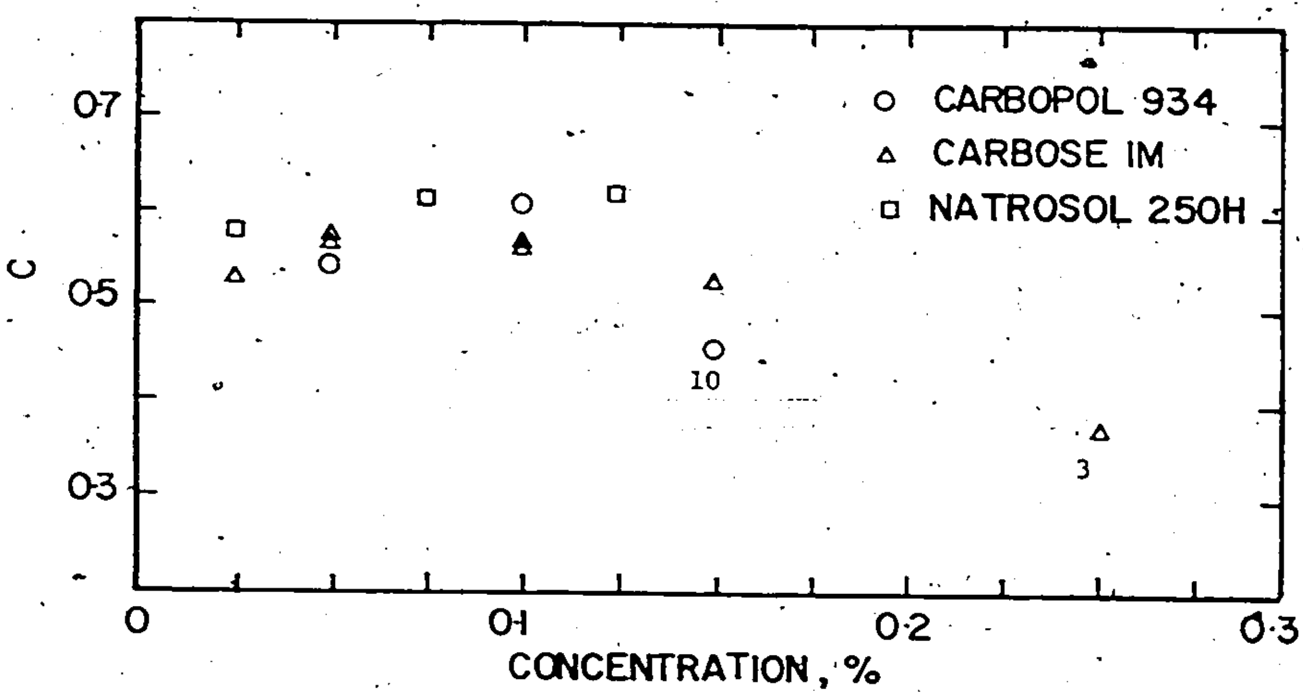


FIG. 6·1 C VS CONCENTRATION

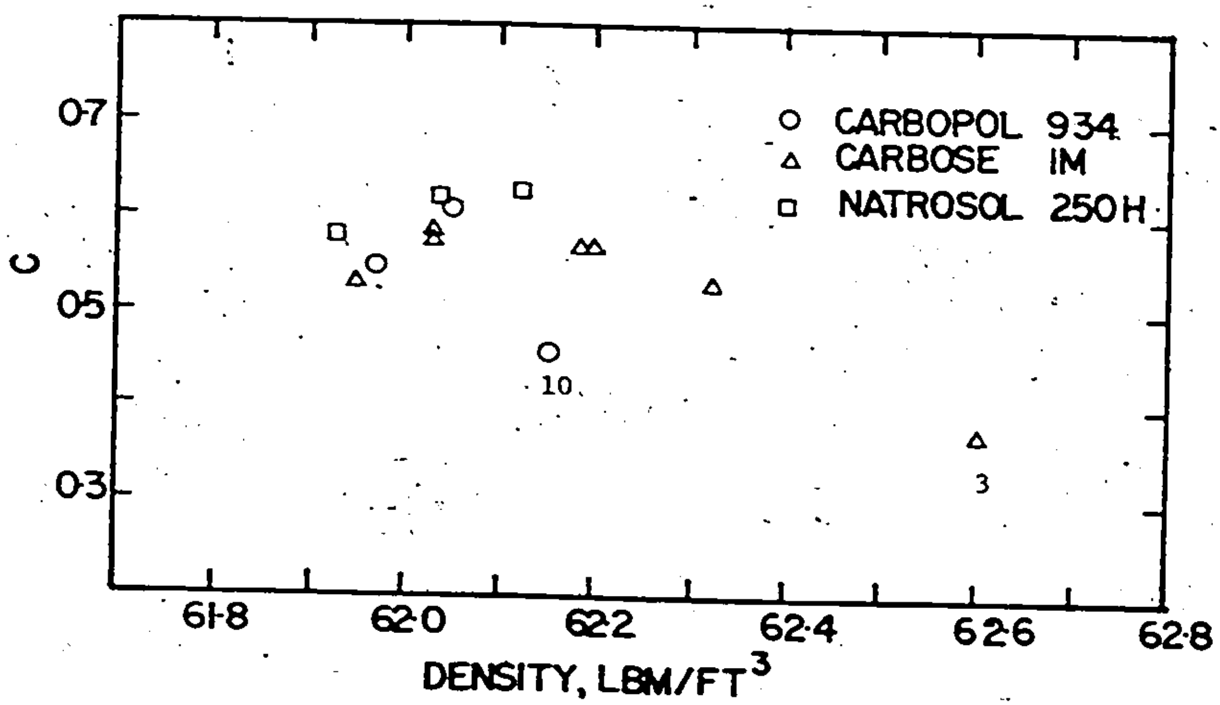


FIG. 6·2 C VS DENSITY

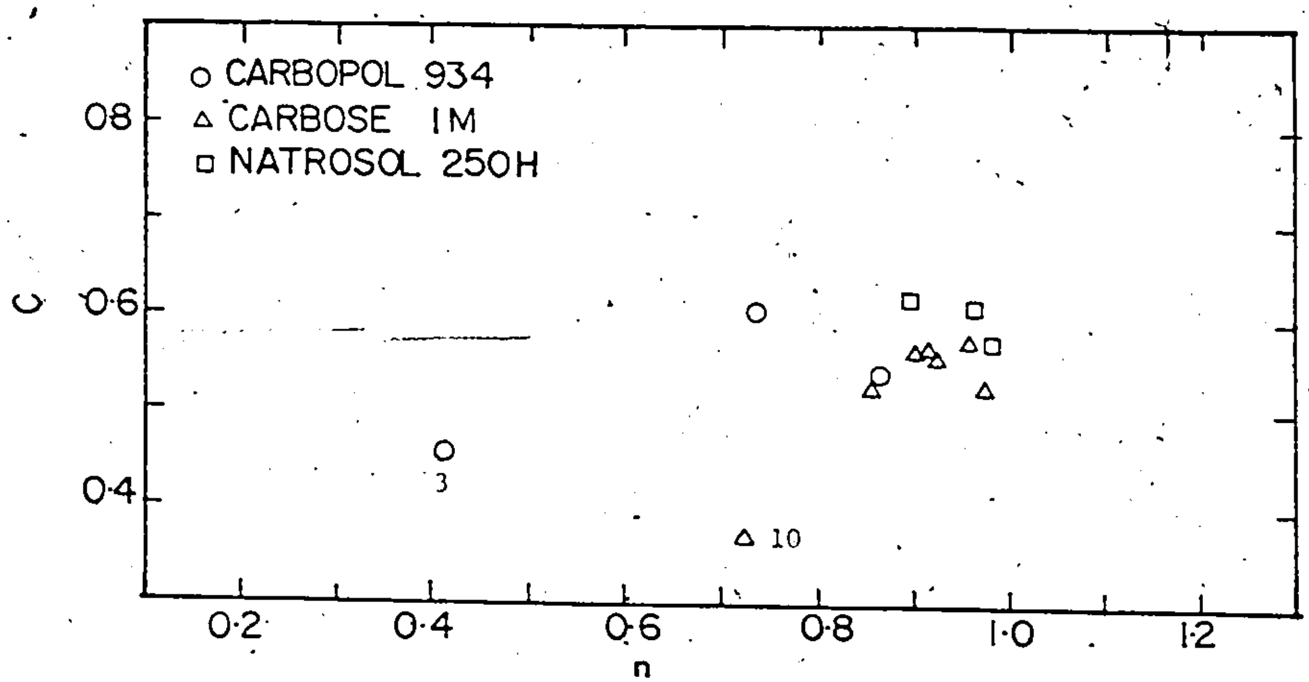


FIG. 6·3 C VS n_{ave}

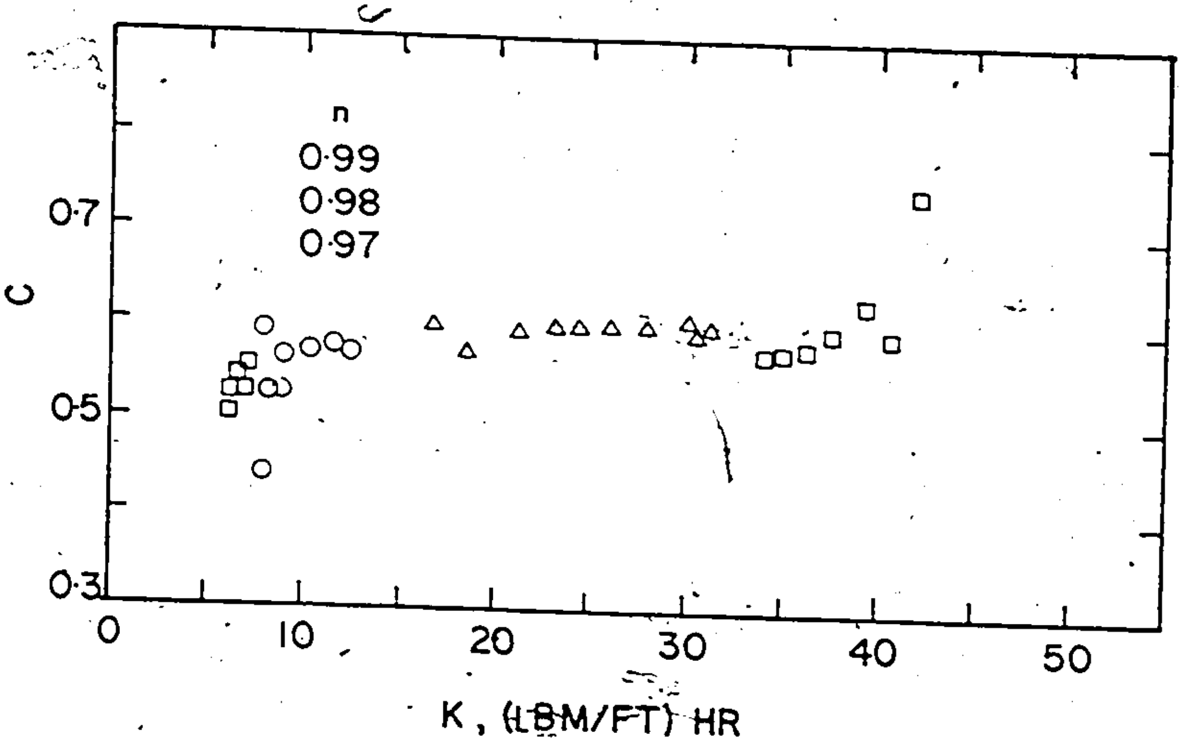


FIG. 6·4 C VS K

Table 6.2 RANGES OF K and n FOR POLYMER SOLUTIONS

Solution	C	Range of K $\frac{lbm}{ft.}(hr)^{2-n}$	Range of n
1	0.544	32 - 42	0.838 - 0.869
2	0.608	610 - 2753	0.663 - 0.769
3	0.454	30714 - 503948	0.294 - 0.518
4	0.530	30 - 50	0.887 - 0.912
5	0.571	32 - 51	0.880 - 0.912
6	0.565	66 - 201	0.909 - 0.919
7	0.578	12 - 35	0.952 - 0.964
8	0.563	46 - 187	0.916 - 0.939
9	0.527	445 - 1088	0.842 - 0.845
10	0.370	9823 - 21814	0.707 - 0.745
11	0.587	10 - 46	0.972 - 0.992
12	0.613	313 - 1644	0.961 - 0.965
13	0.621	5191 - 24163	0.886 - 0.900

data obtained by using cylinders, whose diameters approach that of a wire, as heat sources. A film theory presented by Langmur (23) is used to explain this phenomenon. The explanation is that for a cylinder of very small diameter, like a wire, a thin film of finite thickness surrounds the cylinder and heat is transferred through the film by conduction. Langmur's film theory was studied by Eleenbaas(24, 45), who derived equation (6.1)

$$235\, N_{Nu}^{3}\, e^{-6/N_{Nu}} = N_{Gr} N_{pr} \tag{6.1}$$

Excellent agreement between equation (6.1) and the experimental results was found in fluids such as air, glycerol, water and mercury.

In a contentrated polymer solution, this phenomenon might prevail. The film thickness formed around the sphere is so thick that heat might be transferred through conduction. In other words, the formation of a thick film has simply violated the boundary layer theory that the transfer of momentum and energy has to take place in a very thin layer of film adjacent to the heating surface.

The data obtained from solutions 3 and 10 are therefore judged to be unsuitable for convective heat transfer treatment. The overall correlation would include solutions 1, 2, 4, 5, 6, 7, 8, 9, 11, 12 and 13 only. The correlation that contains solutions 3 and 10 is included just for interest and to see how badly the correlation is affected.

B. Conclusions

In this study, experiments have been carried out to investigate the theoretical equations derived for laminar free convection to both Newtonian and non-Newtonian fluids. The heat transfer medium was a 5 inch diameter copper sphere. The Newtonian fluid studied was water and the non-Newtonian fluids are three different concentrations of Carbopol 934 (0.5%, 1% and 1.5% in acid form), seven Carbose 1M (0.25% to 2.5%) and three Natrosol 250H (0.25% to 1.25%) aqueous solutions. The following results and conclusion were obtained.

a. Newtonian Fluid-Water

Theories of Merk and Prins (49) and Acrivos (1) were confirmed. The following equations are obtained.

With L = radius of the sphere

$$\overline{N_{Nu}} = 0.494\,[N_{Gr} N_{Pr}]^{1/4} \tag{6.2}$$

$$14.7 \le \overline{N_{Nu}} \le 57$$

$$8.8 \times 10^5 \le N_{Gr}\,N_{Pr} \le 3.4 \times 10^8$$

The mean error is 4.5%.

With L = diameter of the sphere,

$$\overline{N_{Nu}} = 0.588\,[N_{Gr}\,N_{Pr}]^{1/4} \tag{6.3}$$

$$29 \le \overline{N_{Nu}} \le 138$$

$$7.1 \times 10^6 \le N_{Gr}\,N_{Pr} \le 2.73 \times 10^9$$

The mean error is 4.5%.

The treatment of physical properties as a funciton of temperature seems to have negligible effect on the theoretical value of C, which is based on constant physical properties.

b. Non-Newtonian Fluids

Acrivos' equation for non-Newtonian fluids is conffrmed with the modified use of the equation. The values of C are higher than the theoretical ones. The variation of physical properties in the correlation does not seem to be the cause of the deviation of C. It could be the intermolecular attractions that affect the C. The C value is slightly dependent on the type of solution. For concentration higher than 1.25% the C value decreases significantly compared to the more diluted solution of the same type. There is no evidence found on the dependency of C on K and n. It therefore is concluded that the decrease in C for more concentrated and thick solution is very probably affected by the film formed on the surface of the heating medium so that heat has to transfer through this layer by conduction.

The overall correlation of equation (3.40) for 0.5% and 1% Carbopol 934, 0.25%, 0.5% and 1.0% Carbose 1M, 0.5%, 1% and 1.5% Carbose 1M (with 0.1% sodium benzoate), and 0.25%, 0.75% and 1.25% Natrosol 250H (with 0.1% Sodium benzoate) is

$$\overline{N_{Nu}} = 0.561\, N_{Gr}^{\frac{1}{2(n+1)}}\, N_{Pr}^{\frac{n}{3n+1}} \qquad (6.4)$$

where

$$2.5 \le N_{Nu} \le 55.5$$

$$7.99 \times 10^{2} \le N_{Gr} N_{Pr} \le 1.33 \times 10^{8}$$

$$0.66 \le n \le 0.99$$

and $10 \le K \le 24163$

The mean error is 5.9%

NOMENCLATURE

A: thermal conductivity apparatus constant, Btu/hr.°F

a: parameter of equation (3.7), $ft^2/lbf\ sec^{-1}$

a_1: parameter of equation (3.9), lbf/ft^2

a_2: parameter of equation (3.10), sec^{-1}

a_3: parameter of equation (3.11), lbf/ft^2

a_4: parameter of equation (3.12), $lbf\ sec^n/ft^2$

Ar: surface area, ft^2

B: thermal conductivity apparatus constant, ft

b: parameter of equation (3.7), $ft^{2\alpha}/sec\ lb_f^{\alpha}$

b_1: parameter of equation (3.9), sec^{-1}

b_2 : parameter of equation (3.10), ft^2/lbf

b_4: parameter of equation (3.12), $lbf\ sec^m/ft^2$

C: parameter of equations (3.40) and (3.49), dimensionless

C': parameter of equation (3.15), dimensionless

C_p: Specific heat per unit mass, Btu/lbm°F

C_1: specific heat per unit mass (3.50) Btu/lbm°F

C_2: specific heat per unit mass (3.50) Btu/lbm°F

C_3: specific heat per unit mass (5.7), Btu/lbm°F

C_4: specific heat per unit mass (5.7), Btu/lbm°F

c: parameter of equations (3.4), $(lb_f/ft^2)^{-n}$

c_2: parameter of equation (3.10), $lbf\cdot sec/ft^2$

f: function in equation (3.35)

f_1: function in equation (2.1)

f_2: function in equation (2.1)

f_3: function in equation (2.2)

g_c: gravitational force or conversion factor

$\bar{h}$: mean heat transfer coefficient, Btu/hr. ft^2°F

K: parameter of equation (3.6), lbm sec^{n-2}/ft or lbf/ft sec^n

k: thermal conductivity coefficient, Btu/hr ft°f

k_o, k_1, k_2: Polynomial coefficients of equation (5.1)

L: characteristic length, inch

L_{eq}: equivalent length, inch

m: parameter of equation (3.3), (3.4), (3.8) and (3.12), dimensionless

n: parameter of equation (3.6), dimensionless

N_{Gr}: Grashof number, dimensionless, defined by eqn. (3.13) or (3.21)

N_{Nu}: local Nusselt number, dimensionless

$\overline{N_{Nu}}$: average Nusselt number, dimensionless

N_{Pr}: Prandtl number, dimensionless, defined by eqn. (3.14) or (3.22)

N_{Pr}': Prandtl number, dimensionless, defined by eqn. (3.51)

N_{Ra}: Rayleigh number, dimensionless

Q: total heat input to sphere, Btu/hr.

q: total heat input to thermal conductivity cell, Btu/hr

r: distance from the axis of symmetry

r_1: dimensionless distance

S: volume fraction of solid in suspension

T: temperature, °F

T_b: bulk temperature of fluid, °F

T_S: surface temperature, °F

u: velocity component along x

Uc: characteristic velocity

u_1: dimensionless velocity

u_2: dimensionless velocity

$\bar{u}_2$: dimensionless velocity

v : velocity component along y

v_1: dimensionless velocity

v_2: dimensionless velocity

x : distance along surface from the leading edge

x_1 dimensionless distance

Y : dependent variable of eqn. (5.1)

y : distance normal to surface

y_1: dimensionless distance

y_2: dimensionless distance

Z : $N_{Gr}^{\frac{1}{2(n+1)}} \quad N_{Pr}^{\frac{n}{3n+1}}$

Z_1: $N_{Gr}^{\frac{1}{2(n+1)}}$

Z_2: $N_{Pr}^{\frac{n}{3n+1}}$

GREEK LETTER

α: parameter in equation (3.7), dimensionless

β: thermal expansion coefficient, $°F^{-1}$

ε: angle between the normal to the surface and the direction of the force of gravity

η: similarity variable defined by equation (3.34)

θ: dimensionless temperature,

λ: parameter of equation (3.5), $(lbf/ft^2)^{0.2}$

μ: viscosity of Newtonian fluid, lbm/ft sec

μ_∞ : viscosity at infinite shear in equation (3.8), lbm/ft sec.

μ_L : viscosity of liquid suspending medium, lbm/ft sec

μ_o: viscosity at zero shear in equations (3.8) and (3.11)

ξ: coefficient of rigidity in equation (3.2), lbm/ft sec

ξ_o: coefficient of rigidity at zero shear, lbm/ft sec

ξ': constant, $(lbf)^{m-1}$ lbm ft $^{1-2m}$ sec $^{-1}$

ρ: density of fluid, lbm/ft^3

ρ_∞: density of fluid at T_∞

τ_y: yield stress, lbf/ft^2

τ_{yx}: shearing stress, lbf/ft^2

χ: dimensionless distance defined by equation (3.42)

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APPENDIX I

NOTES ON TEST FLUIDS

TEST FLUIDS

A. Selection of Test Fluids

The only Newtonian fluid employed was water, which was selected because of the availability of extensive physical property data and previous experimental findings on similar equipment (2, 6). The data obtained for water also served as a calibration for the equipment.

The non-Newtonian solutions were selected for the following main reasons:

1. The shearing stress and shearing rate relationship followed the power-law.
2. They must be able to exhibit a comparatively wider range of non-Newtonian behaviour in the low shearing rate range which is the characteristic of free convection.
3. Their rheological properties are time independent and free from any viscoelastic effects.
4. Their physical properties are stable.

The rheological properties of a number of aqueous solutions of polymers had been testd. They are:

1. Carbopol 934 (Carboxy Vinyl polymer), manufactured by B. F. Goodrich Chemical Co.
2. Carbose IM (Sodium Carboxy Methylcellulose), manufactured by Wyandotte Chemical Corp., Industrial Chemical Group.
3. Carbowax (Polyethylene Glycols), manufactured by Union Carbide.
4. Elvanol (Polyvinyl Alcohol), manufactured by Du Pont.
5. Methocel HG 15 (Methyl-Cellulose), manufactured by Dow Chemical Co.
6. Natrosol 250H (Hydroxyethyl Cellulose), manufactured by Hercules Inc.
7. PVP K90 (Polyvinylpyrolidone), manufactured by General Aniline and Film Corp.

Out of the above seven polymers, only Carbopol 934, Carbose 1M and Natrosol 250H were found to be suitable for the present study. The following solution concentrations were used. (All percentage indicated are on a weight basis).

Carbopol 934: 0.5, 1 and 1.5%

Carbose 1M : 0.25, 0.5 and 1%

0.5, 1.5 and 2.5% (with 0.1% Sodium Benzoate).

Natrosol 250H: 0.25, 0.75 and 1.25% (with 0.1% Sodium Benzoate).

Carbopol 934 was used in acid form with pH equal to about 3. It has been observed that the addition of 0.1% Sodium Benzoate, which worked as a germicidal agent to preserve the solution from being attached by bacteria, had no significant effect on the physical properties of the solutions. Also it has been observed that the apparent viscosity (or shearing stress) of these solutions decreases about 1.5% for every 24 hour period. Since it usually takes about three days to complete a series of run for each solution, the possible maximum error caused by the decrease in apparent viscosity of the solution is about 5%. Extra precaution was therefore taken in which heat transfer and physical property data (especially the rheological properties) were determined during the same time period.

B. Preparation of Solutions

a. Newtonian Fluid

The only treatment required for distilled water was deaeration. Low pressure steam passing through a copper coil was used to raise the temperature of the water in the mixing tank to about 150°F. Because of the decrease of solubility of air in fluids at elevated temperature, most air will escape in the form of bubbles. The solution was gently mixed while being heated. The mixing helped to achieve uniform heating and release bubbles formed on the heating coil. The solution was allowed to cool to room temperature after being heated at about 150°F for about 30 minutes.

b. Non-Newtonian Fluids

For the non-Newtonian solutions, the precautions suggested by the manufacturer (10, 11, 12) were followed. The aqueous solutions of the three types of polymer are prepared basically in the same way. About 80 gallons of water was placed in the mixing tank. It was mixed vigorously with a 1/4 hp. Lightnin Mixer, the water soluble polymer was then added slowly in order to facilitate dispersion, and avoid the formation of lumps. The mixing rate had to be adjusted in order to get the best mixing effect during the dissolution process. For Carbose IM, about 24 hours was needed for complete dissolution, while for Carbopol 934 and Natrosol 250H the dissoluions were much faster. After complete dissolution deaeration was carried out. Deaeration was necessary in order to prevent the formation of bubbles on the surface of the sphere due to the excess air being liberated in the fluid on heating up.

C. Physical Properties

The physical properties of water are available in (30). All physical properties, except the heat capacity were experimentally determined for all the polymer solutions. Details are given in Chapter IV. The heat capacity was assumed to be equal to that of water and treated as unity based on the work of St. Pierre (81) and Vaughn (87).

APPENDIX II

DENSITY AND
THERMAL EXPANSION COEFFICIENT

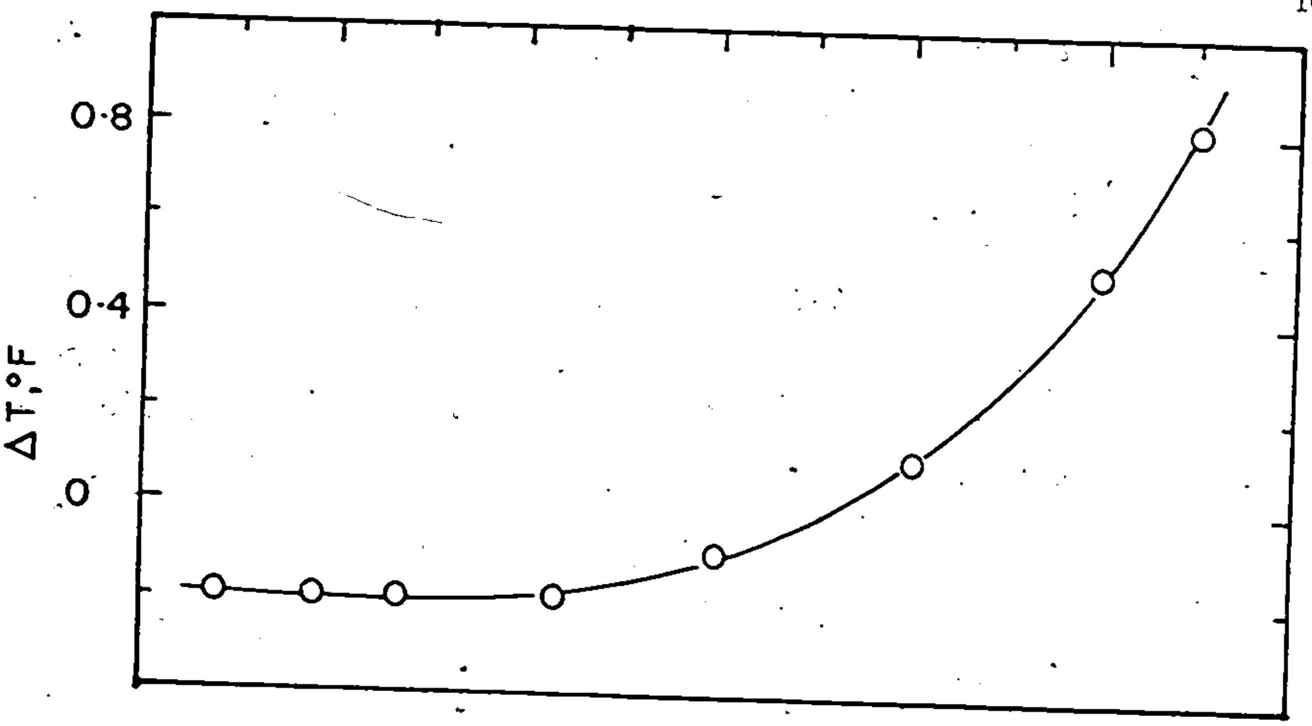


FIG. A2·1 PYCNOMETER TEMP. VS BATH TEMPERATURE

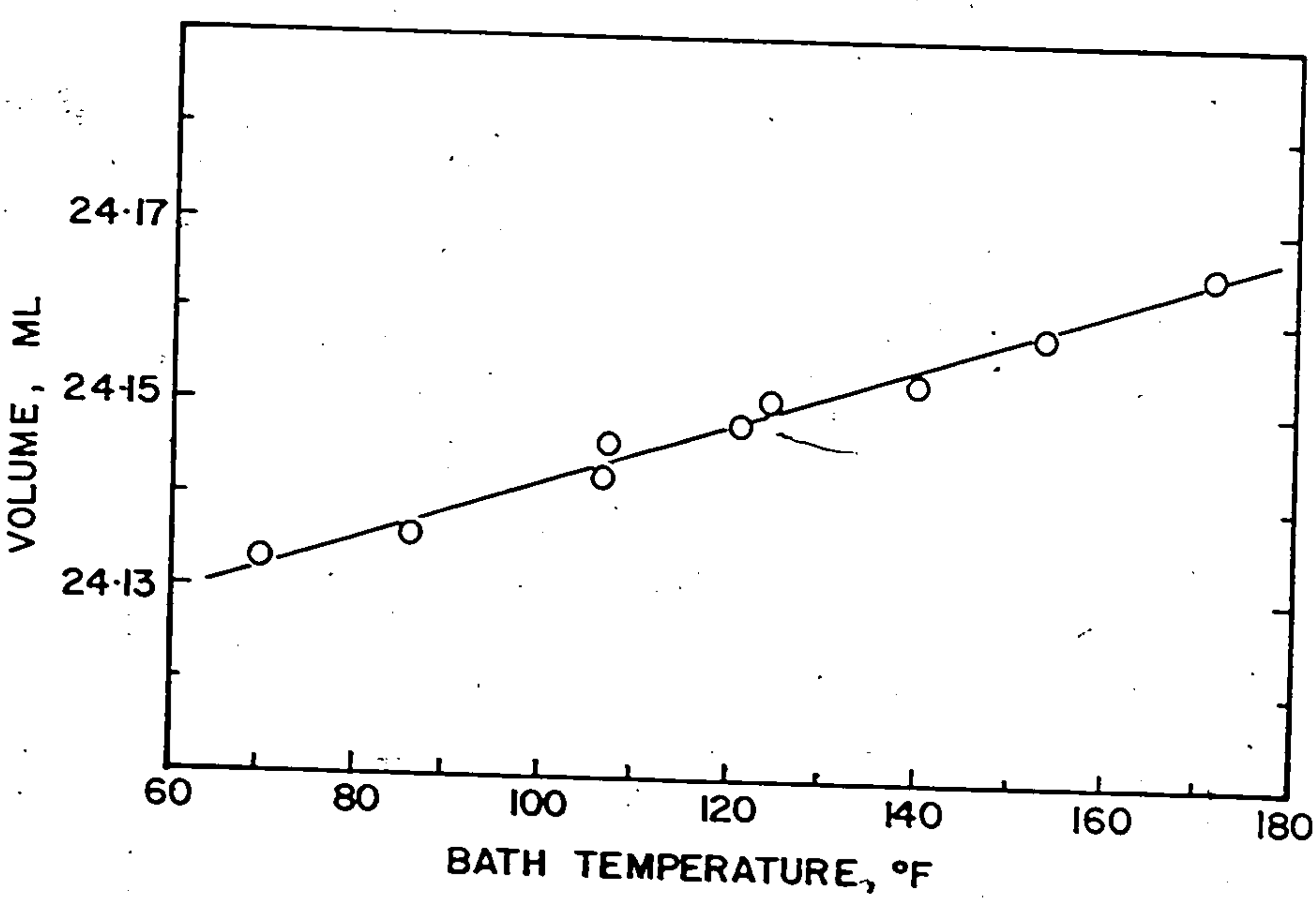


FIG. A2·2 VOLUME OF PYCNOMETER

Table A2.1
Pycnometer Temp. vs Bath Temp.

Bath Temp. °F	Fluid Temp. °F	ΔT* °F
67.2	67.4	-0.2
77.7	77.9	-0.2
86.6	86.8	-0.2
102.8	103	-0.2
121.6	121.7	-0.1
140.8	140.7	+0.1
159.4	158.9	+0.5
171.6	170.8	+0.8

*ΔT=bath temp. - bottle temp.

Table A2.2
Volume of Pyenometer

Bath Temp. °F	Volume ml
69.4	24.1327
85.6	24.1355
106.2	24.1424
106.5	24.1467
120.7	24.1479
123.4	24.1512
139.8	24.1527
153.5	24.1582
165.4	24.1651

TABLE A2.3

ρ AND β FOR CARBOPOL 934 SOLUTIONS

0.5%			1%			1.5%		
T °F	ρ LBM/FT3	β*10^4 °F-1	T °F	ρ LBM/FT3	β*10^4 °F-1	T °F	ρ LBM/FT3	β*10^4 °F-1
67.8	62.437	1.173	67.8	62.533	1.290	67.8	62.615	1.228
76.8	62.351	1.451	76.8	62.457	1.516	76.8	62.538	1.494
85.8	62.269	1.710	85.8	62.370	1.735	85.8	62.451	1.741
94.8	62.175	1.948	94.8	62.261	1.949	94.8	62.340	1.971
103.8	62.051	2.167	103.8	62.144	2.156	103.8	62.226	2.182
112.8	61.927	2.365	112.8	62.017	2.356	113.2	62.096	2.3 4
121.9	61.788	2.546	121.9	61.882	2.552	121.9	61.959	2.552
131.0	61.639	2.706	131.0	61.732	2.743	131.0	61.807	2.710
140.1	61.483	2.846	140.1	61.583	2.926	140.1	61.661	2.849
149.2	61.324	2.964	148.3	61.408	3.097	149.2	61.490	2.969

TABLE A2.4

ρ AND β FOR CARBOSE 1M SOLUTIONS

0.25%			0.5%			1%		
T °F	ρ LBM/FT3	β*10^4 °F-1	T °F	ρ LBM/FT3	β*10^4 °F-1	T °F	ρ LBM/FT3	β*10^4 °F-1
66.5	62.424	1.083	67.9	62.488	1.209	67.8	62.661	1.227
77.2	62.336	1.427	76.3	62.415	1.459	76.8	62.581	1.484
89.0	62.220	1.775	85.8	62.326	1.679	85.8	62.488	1.725
101.5	62.072	2.107	94.3	62.226	1.898	94.8	62.394	1.949
111.7	61.925	2.351	103.3	62.117	2.105	103.3	62.276	2.158
116.5	61.860	2.456	112.8	61.986	2.302	112.8	62.148	2.350
128.5	61.671	2.697	121.9	61.856	2.490	121.9	62.003	2.528
138.5	61.489	2.872	131.0	61.711	2.667	131.0	61.867	2.688
154.3	61.265	3.098	140.1	61.560	2.833	140.1	61.706	2.832
165.6	60.987	3.223	149.2	61.392	2.988	149.2	61.545	2.959

Table A2.5

ρ and β for Carbose IM Solutions (with 0.1% Sodium Benzoate)

0.5%			1%			1.5%			2.5%		
Temp. °F	ρ lb_m/ft^3	$\beta \times 10^4$ °F^{-1}	Temp. °F	ρ lb_m/ft^3	$\beta \times 10^4$ °F^{-1}	Temp. °F	ρ lb_m/ft^3	$\beta \times 10^4$ °F^{-1}	Temp. °F	ρ lb_m/ft^3	$\beta \times 10^4$ °F^{-1}
67.8	62.521	1.260	68.0	62.662	1.322	67.8	62.798	1.240	68.0	63.116	1.276
76.8	62.446	1.493	76.8	62.581	1.540	76.8	62.727	1.497	76.8	63.037	1.515
85.8	62.352	1.717	85.8	62.493	1.756	85.8	62.633	1.738	85.8	62.944	1.748
94.8	62.252	1.932	94.8	62.382	1.963	94.8	62.532	1.963	94.8	62.837	1.968
103.8	62.136	2.138	103.8	62.269	2.163	103.8	62.404	2.172	103.8	62.731	2.176
112.8	62.015	2.334	112.8	62.143	2.356	112.8	62.281	2.365	112.8	62.590	2.372
121.9	61.877	2.524	121.9	62.007	2.543	121.9	62.148	2.543	121.9	62.452	2.558
131.0	61.726	2.704	131.0	61.857	2.722	131.0	61.998	2.705	131.0	62.306	2.732
140.1	61.573	2.874	140.1	61.693	2.894	140.1	61.843	2.849	140.1	62.143	2.893
149.2	61.407	3.036	149.2	61.533	3.058	149.2	61.674	2.978	149.2	61.977	3.042

TABLE A2.6

ρ AND β FOR NATROSOL 250H SOLUTIONS (0.1 % SODIUM BANZOATE ADDED)

0.25 %			0.75 %			1.25 %		
T °F	ρ LBM/FT³	β*10⁴ °F⁻¹	T °F	ρ LBM/FT³	β*10⁴ °F⁻¹	T °F	ρ LBM/FT³	β*10⁴ °F⁻¹
67.8	62.405	1.165	67.8	62.059	1.249	67.8	62.591	1.235
76.8	62.327	1.441	76.8	62.431	1.500	76.8	62.511	1.479
85.8	62.243	1.608	85.8	62.333	1.738	85.8	62.426	1.714
94.8	62.141	1.735	94.8	62.237	1.960	94.8	62.319	1.939
103.8	62.022	2.153	103.8	62.123	2.169	103.8	62.208	2.155
112.8	61.905	2.351	112.8	61.994	2.362	112.8	62.083	2.361
121.9	61.766	2.532	121.9	61.857	2.544	121.9	61.937	2.561
131.0	61.611	2.693	131.0	61.705	2.710	131.0	61.789	2.751
140.1	61.460	2.833	140.1	61.554	2.861	140.1	61.636	2.932
149.2	61.301	2.953	149.2	61.388	2.998	149.2	61.461	3.103

APPENDIX III

THERMAL CONDUCTIVITY

THERMAL CONDUCTIVITY OF LIQUIDS (65)

A. Theory of the Method of Measurement

According to the basic equation for heat conductivity, the amount of heat q per unit time flowing through a layer of known geometrical dimensions, is proportional to the temperature difference of the two interfaces of the layer and to the thermal conductivity k of the medium:

$$q=B\ k\ \Delta T \tag{III.1}$$

where B is the proportionality factor which is dependent upon the geometrical dimensions of the layer and which may be considered as an apparatus constant. Equation (III.1) is also valid in cylindrical arrangement, for which, however, B assumes different physical importance than for plain layers. Applying this equation to the present experimental arrangement, it is necessary to compile the overall heat flow from the heating block by adding the various parallel portions of the different parts of the liquid layers and the comparatively small losses through the centering stainless steel rod. Considering further that the thermal conductivity of stainless steel is 30 times higher than that of water, it may be assumed with close approximation that the temperature at the two interfaces of the liquid layer may be considered uniform, even with locally slightly different densities of heat flow. If separation is made of the portions of the liquid layer and the loss through the centering rod, and considering that the latter is independent of the kind of liquid under test, it follows that there is a linear relationship between the quotient $q/\Delta T$ and the thermal conductivity of the liquid:

$$q/\Delta T=A+B\ k \tag{III.2}$$

In this equation, which may be assumed valid for the present experimental arrangement (see part B), ΔT represents the temperature difference between the surface of the heating block and the corresponding inner surface of the cooling block. If the value of ΔT is less than 1^oC no convection would occur for a liquid layer of 1mm. The linear part B k represents essentially the heat flow through the liquid and the constant A, the correction for heat loss.

B. Apparatus

The set up of the apparatus was somewhat modified from the original design (see Fig. 3A.1). The air surrounding the vacuum flask which is contained in a wooden box was controlled to about 85^oF in order to reduce the effect of the fluctuation of room temperature on the steady state operation of the equipment. The test chamber was first filled by suction through the side arm and later by siphon effect caused by raising the flask that contained the test solution until a proper position was reached. The level of the solution was always the same so that the hydrostatic pressure in the test chamber remained unchanged. A more detail of the test chamber is given in Fig. A3.2.

C. Procedure

The procedure for measuring the thermal conductivity of a solution was as follows:

(a). The sample solution was deaerated.

(b). The test chamber was flushed with the test sample and filled up carefully.

(c). The apparatus was set up as shown in Fig. A3.1. The temperature in the box was controlled to about 85^oF.

(d). The system was energized with a preselected heat input to the heating element and compensating heater.

(e). The sink was filled with ice water mixture to a fixed level.

(f). After steady state was reached, which usually could be achieved in five or six hours by a skilled operator, the voltage and current to the heating element, temperature of the heating and cooling surfaces were recorded.

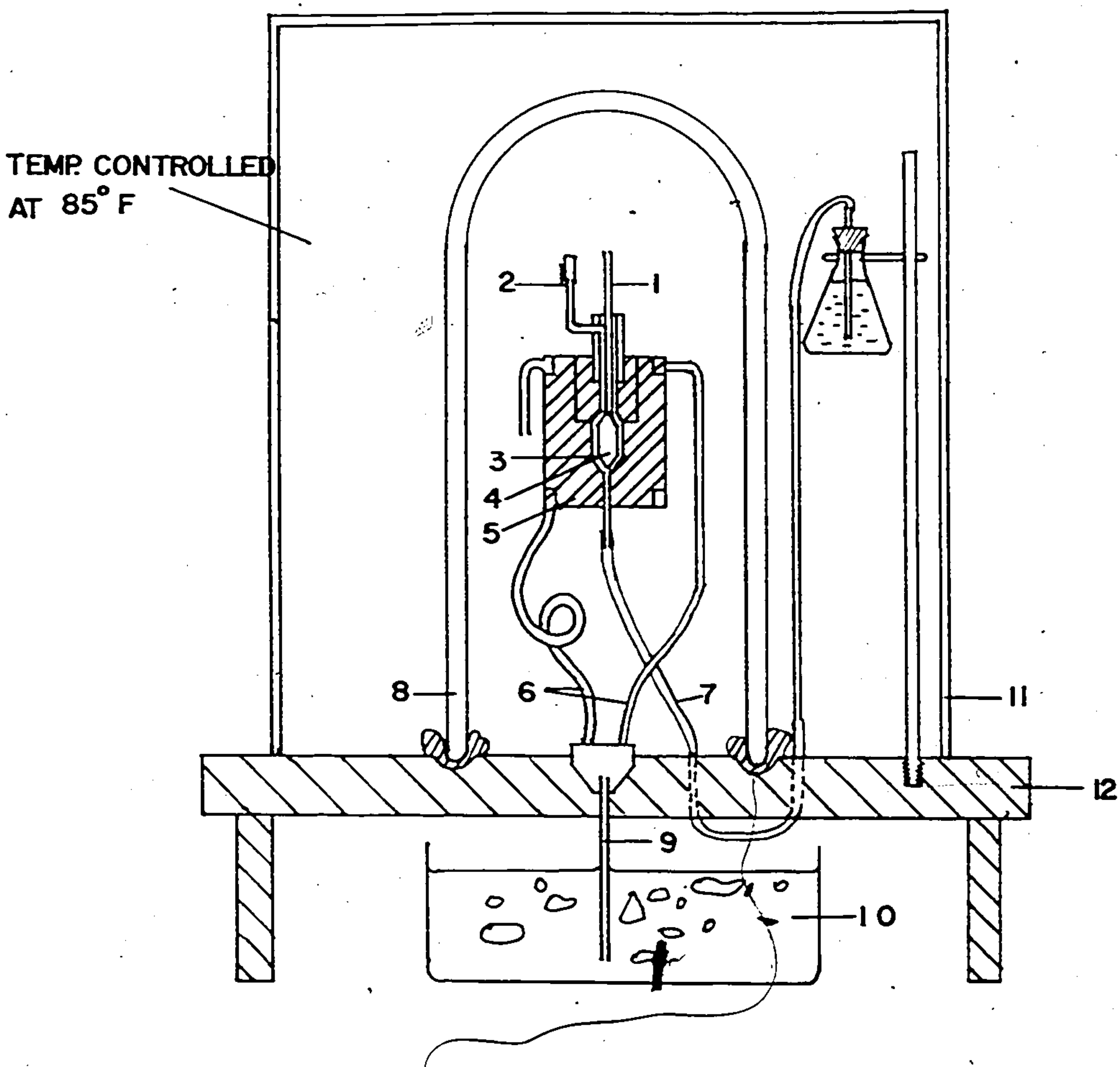


FIG. A3-1 THERMAL CONDUCTIVITY APPARATUS SET UP

THERMAL CONDUCTIVITY APPARATUS DETAIL

Symbol	Description
1	Supporting rod
2	Side arm
3	Annular liquid gap
4	Heating block
5	Cooling block
6	Copper cooling coil (12)
7	Tygon tube
8	Vacuum flask
9	Cooling rod
10	Ice water mixture
11	Wooden box
12	Plywood sheet

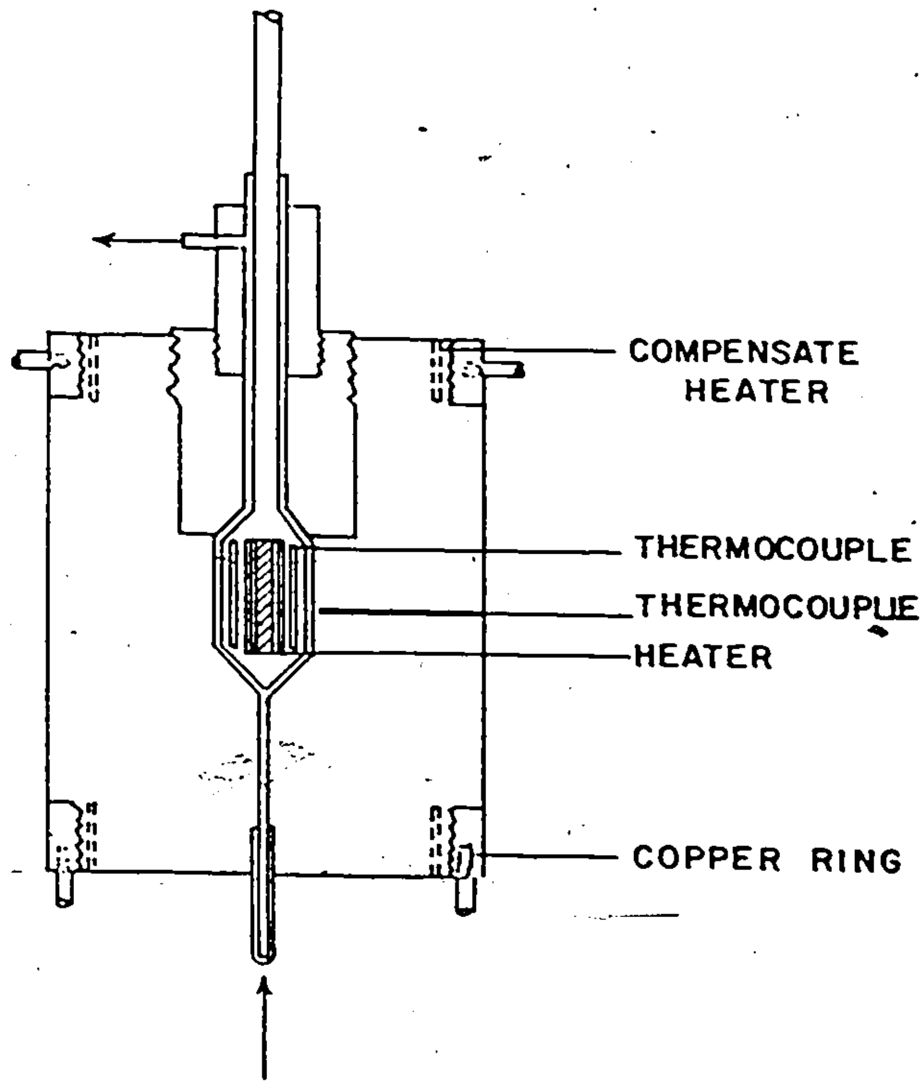
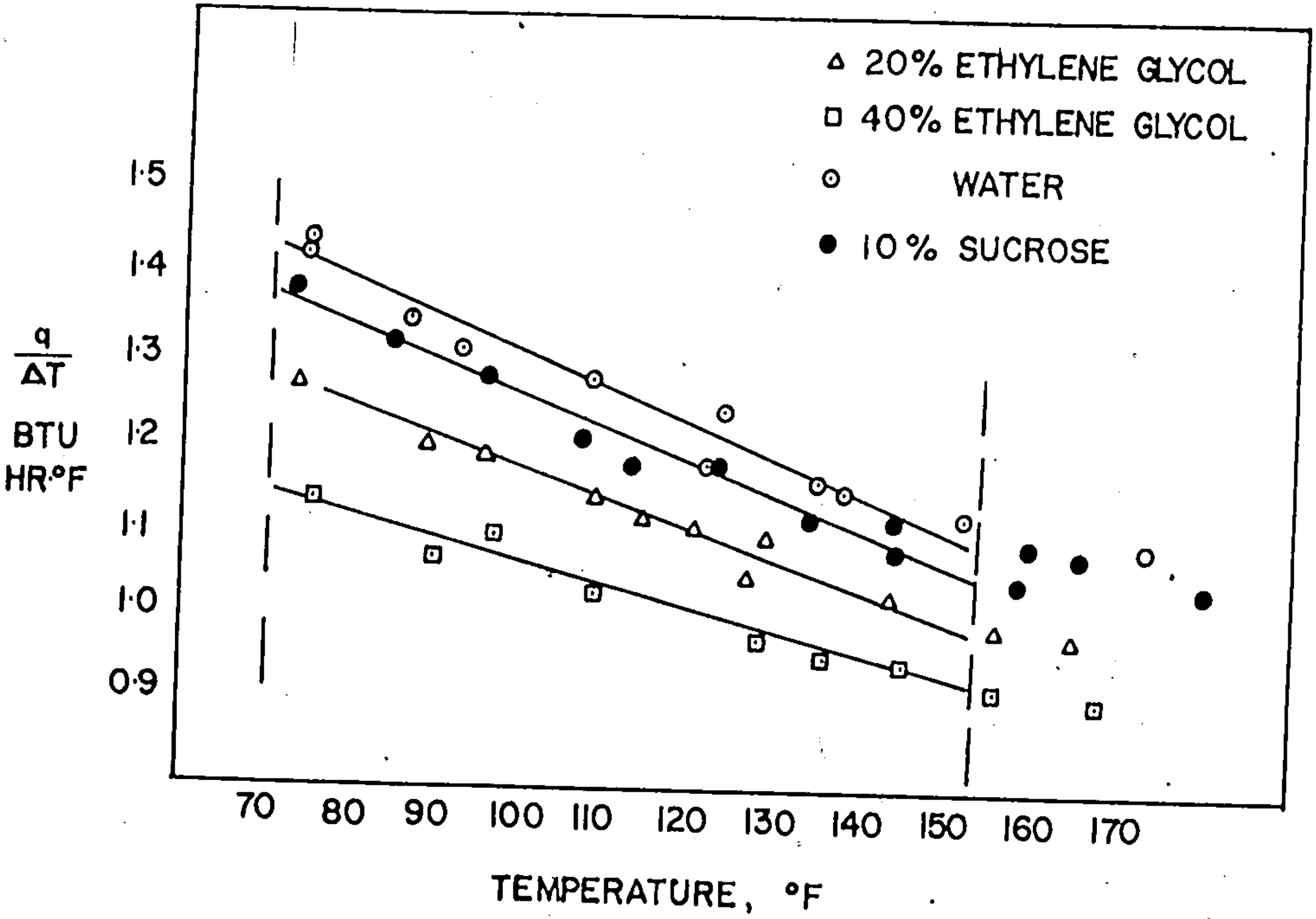


FIG.A32 DETAILS OF THERMAL CONDUCTIVITY APPARATUS (g). Steps (d) to (f) were repeated after increasing heat input to the compensating heater for each subsequent run to increase the average temperature of the system. The average temperature was the arithmatic mean temperature of the heating and cooling surfaces.



FIG· A3·3

CALIBRATION DATA FOR THERMAL CONDUCTIVITY APPARATUS·

TABLE A3.1
q / ΔT VS T – WATER

RUN	T °F	ΔT °F	q / T BTU/HR °F
1	74.1	1.366	1.418
2	74.5	1.355	1.432
3	86.1	1.465	1.347
4	92.4	1.508	1.308
5	107.9	1.568	1.273
6	121.6	1.633	1.175
7	123.3	1.612	1.237
8	134.5	1.722	1.158
9	137.4	1.718	1.137
10	151.7	1.766	1.116
11	173.5	1.817	1.096

TABLE A3.2
q / ΔT VS T – 20 % ETHYLENE GLYCOL

RUN	T °F	ΔT °F	q / ΔT BTU/HR °F
1	73.6	1.475	1.268
2	88.5	1.556	1.199
3	95.3	1.643	1.183
4	108.7	1.657	1.138
5	114.1	1.660	1.115
6	120.5	1.668	1.103
7	126.2	1.808	1.047
8	128.7	1.778	1.031
9	143.5	1.840	1.025
10	155.7	1.880	0.988
11	164.3	1.919	0.978

TABLE A3.3
q/ ΔT VS T – 40 % ETHYLENE GLYCOL

RUN	T °F	ΔT °F	q / ΔT BTU/HR °F
1	75.4	1.694	1.136
2	89.6	1.751	1.069
3	96.5	1.759	1.094
4	108.5	1.829	1.026
5	127.9	1.969	0.972
6	135.2	1.996	0.950
7	144.5	2.017	0.948
8	156.4	2.058	0.917
9	167.7	2.133	0.906

TABLE A3.4
q/ΔT VS T – 10 % SUCROSE

RUN	T °F	ΔT °F	q/ ΔT BTU/HR °F
1	72.9	1.413	1.374
2	84.5	1.445	1.316
3	95.1	1.505	1.277
4	106.9	1.552	1.202
5	122.2	1.637	1.179
6	133.2	1.727	1.114
7	143.2	1.778	1.108
8	143.7	1.722	1.116
9	158.1	1.830	1.031
10	159.0	1.821	1.033
11	165.3	1.838	1.071
12	179.9	1.912	1.032

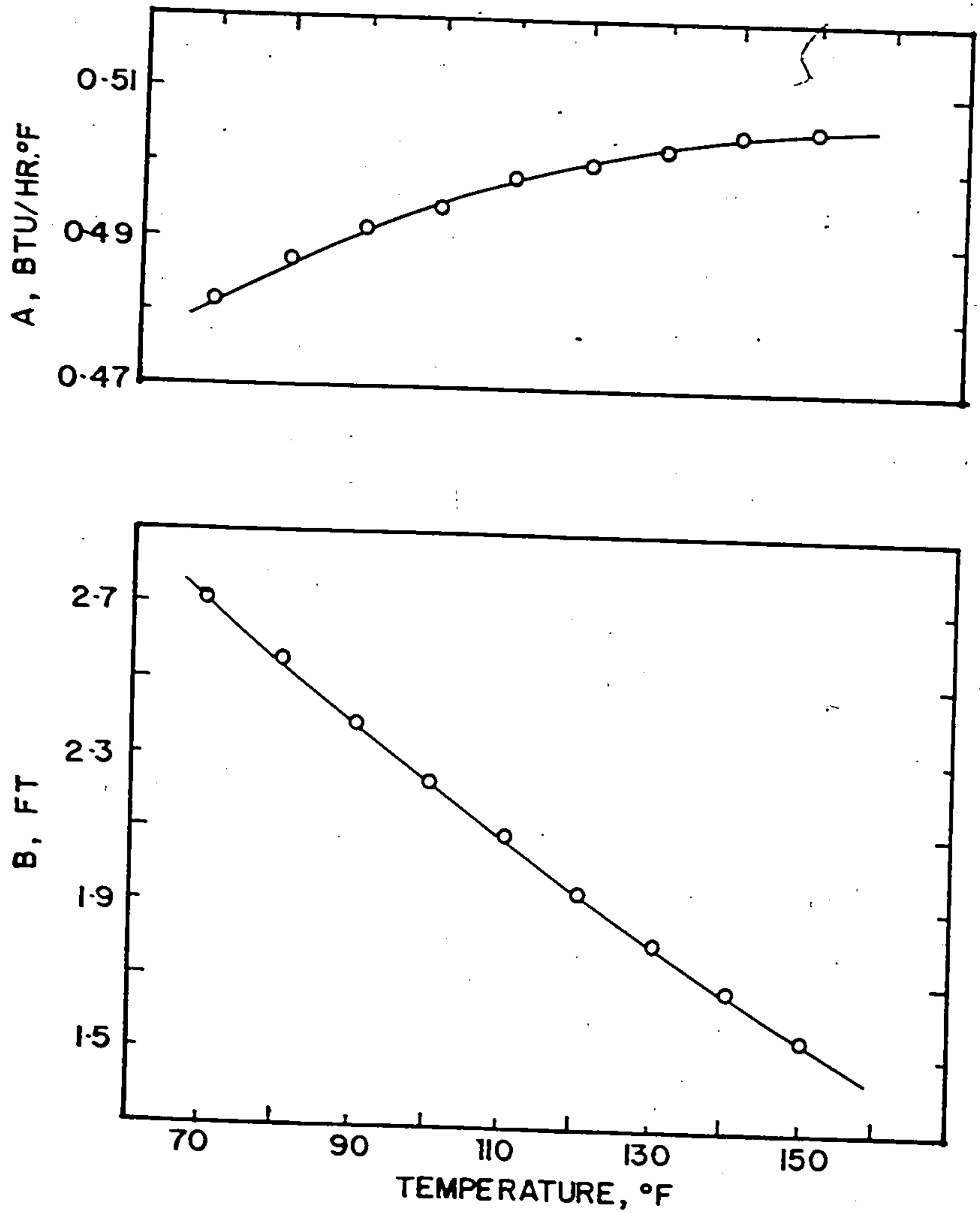


FIG. A3-4 THERMAL CONDUCTIVITY APPARATUS CONSTANTS

Table A3.5 Thermal Conductivity Apparatus Constants

T °F	A Btu/hr °F	B ft
70	0.4819	2.7009
80	0.4871	2.5439
90	0.4914	2.3856
100	0.4949	2.2339
110	0.4985	2.0854
120	0.5005	1.9445
130	0.5026	1.8065
140	0.5048	1.6172
150	0.5055	1.5435

TABLE A3.6

PREDICTIONS ON k FOR CALIBRATION FLUIDS

	WATER			20% ETHYLENE GLYCOL			40% ETHYLENE GLYCOL		
T °F	BTU/HR.FT.°F		ERROR %	BTU/HR.FT.°F		ERROR %	BTU/HR.FT.°F		ERROR %
70	0.348	0.347	0.29	0.296	0.295	0.34	0.245	0.245	0
80	0.352	0.353	-0.28	0.299	0.298	0.34	0.248	0.248	0
90	0.356	0.358	-0.56	0.301	0.303	-0.66	0.250	0.250	0
100	0.361	0.362	-0.28	0.304	0.306	-0.65	0.253	0.253	0
110	0.365	0.366	-0.27	0.307	0.309	-0.65	0.255	0.256	-0.39
120	0.370	0.370	0	0.309	0.313	-1.28	0.258	0.258	0
130	0.374	0.373	0.27	0.312	0.316	-1.28	0.261	0.261	0
140	0.377	0.377	0.53	0.314	0.318	-1.26	0.264	0.263	0.38
150	0.384	0.380	1.05	0.317	0.321	-1.25	0.267	0.266	0.38
MEAN ERROR			0.39			0.78			0.13

Table A3.7

Prediction on k for 10% Sucrose Soln.

T °F	Cal. k Btu/hr.ft.°F	k Btu/hr.ft.°F	Error %
78	0.332	0.329	+0.91
140	0.361	0.358	+0.84

TABLE A3.8

q/ΔT VS T – CARBOPOL 934 SOLN.

RUN	T °F	ΔT °F	q/ΔT BTU/HR°F	CONC. %
1	73.6	1.343	1.434	
2	79.6	1.365	1.394	
3	94.6	1.498	1.302	
4	107.7	1.530	1.253	0.5
5	121.8	1.597	1.211	
6	129.3	1.703	1.145	
7	144.1	1.730	1.116	
1	72.9	1.380	1.292	
2	79.3	1.430	1.235	
3	87.3	1.400	1.334	
4	96.0	1.440	1.324	1.0
5	109.2	1.553	1.243	
6	114.1	1.563	1.204	
7	129.5	1.660	1.153	
9	147.6	1.760	1.096	
1	82.9	1.383	1.332	
2	96.8	1.425	1.307	
3	101.6	1.448	1.281	
4	112.9	1.552	1.192	1.5
5	123.7	1.535	1.196	
6	133.4	1.668	1.111	
7	153.0	1.740	1.058	

TABLE A3.9

$q/\Delta T$ VS T - CARBOSE IM SOLN.

RUN	T °F	ΔT °F	$q/\Delta T$ BTU/HR °F	CONC.
1	73.1	1.325	1.402	
2	73.6	1.190	1.419	
3	73.7	1.295	1.419	
4	77.0	1.333	1.328	
5	82.6	1.378	1.342	
6	97.7	1.519	1.275	0.25
7	99.8	1.518	1.283	
8	109.0	1.595	1.217	
9	121.1	1.603	1.199	
10	135.1	1.717	1.101	
12	153.7	1.773	1.107	
1	73.7	1.412	1.390	
2	78.6	1.410	1.393	
3	87.1	1.462	1.338	
4	95.0	1.512	1.270	
5	109.8	1.548	1.235	
6	110.6	1.568	1.237	
7	113.1	1.600	1.219	0.5
8	119.0	1.473	1.174	
9	120.5	1.440	1.207	
10	129.2	1.550	1.134	
11	130.2	1.474	1.131	
12	141.3	1.750	1.094	
13	142.2	1.773	1.079	
1	72.5	1.323	1.398	
2	79.5	1.409	1.322	
3	94.0	1.435	1.291	
4	96.4	1.450	1.277	
5	111.6	1.535	1.207	
6	111.9	1.550	1.201	1.0
7	131.8	1.610	1.134	
8	133.5	1.615	1.131	
9	139.6	1.653	1.120	
10	143.8	1.663	1.108	
11	145.9	1.715	1.090	
12	154.5	1.758	1.064	

TABLE A3.10
$q/\Delta T$ VS T – CARBOSE 1% SOLN.
(WITH 0.1 SODIUM BENZOATE)

RUN	T °F	T °F	$q/\Delta T$ BTU/HR°F	CONC. %
1	73.0	1.385	1.354	
2	82.8	1.345	1.257	
3	95.1	1.460	1.305	
4	110.4	1.518	1.211	0.5
5	122.6	1.560	1.135	
6	132.4	1.648	1.129	
7	147.1	1.733	1.052	
1	74.5	1.360	1.415	
2	81.0	1.390	1.381	
3	97.8	1.430	1.296	
4	107.8	1.523	1.258	1.0
5	124.3	1.558	1.194	
6	135.4	1.725	1.116	
7	146.7	1.760	1.075	
1	74.3	1.325	1.394	
2	81.6	1.347	1.367	
3	97.1	1.420	1.297	
4	110.4	1.500	1.202	1.5
5	125.9	1.562	1.147	
6	133.5	1.655	1.145	
7	145.9	1.717	1.075	
1	74.0	1.338	1.416	
2	83.4	1.420	1.338	
3	95.5	1.483	1.289	
4	110.6	1.548	1.235	2.5
5	122.3	1.588	1.205	
6	133.9	1.675	1.145	
7	149.9	1.760	1.067	

TABLE A3.11

q / ΔT VS T - NATROSOL 250H SOLN.

RUN	T °F	T °F	q / ΔT BTU/HR °F	CONC. %
1	72.7	1.348	1.404	
2	83.8	1.422	1.237	
3	98.5	1.468	1.[illegible]1	
4	111.4	1.545	1.221	0.25
5	123.4	1.585	1.2[illegible]4	
6	136.5	1.678	1.140	
7	151.4	1.760	1.[illegible]45	
1	73.4	1.323	1.[illegible]89	
2	83.7	1.400	1.[illegible]51	
3	97.2	1.445	1.[illegible]35	
4	112.9	1.560	1.212	0.75
5	123.8	1.595	1.208	
6	137.3	1.702	1.117	
7	156.1	1.790	1.053	
1	74.3	1.342	1.425	
2	84.1	1.390	1.[illegible]56	
3	98.6	1.505	1.232	
4	112.7	1.542	1.238	1.25
5	123.5	1.572	1.211	
6	138.3	1.650	1.141	
7	151.5	1.702	1.101	

TABLE A3.12

THERMAL CONDUCTIVITY OF POLYMER SOLUTIONS

T, °F	k, BTU/HR.FT.°F			
	WATER	CARBOSOL 1M	CARBOPOL 934	NATROSOL 250H
70	0.347	0.3429	0.3472	0.3455
80	0.3525	0.3449	0.3506	0.3491
90	0.3575	0.3478	0.3527	0.3527
100	0.362	0.3504	0.3554	0.3563
110	0.366	0.3528	0.3579	0.3599
120	0.370	0.3550	0.3502	0.3636
130	0.3735	0.3750	0.3624	0.3672
140	0.377	0.3585	0.3642	0.3707
150	0.380	0.3596	0.3656	0.3741

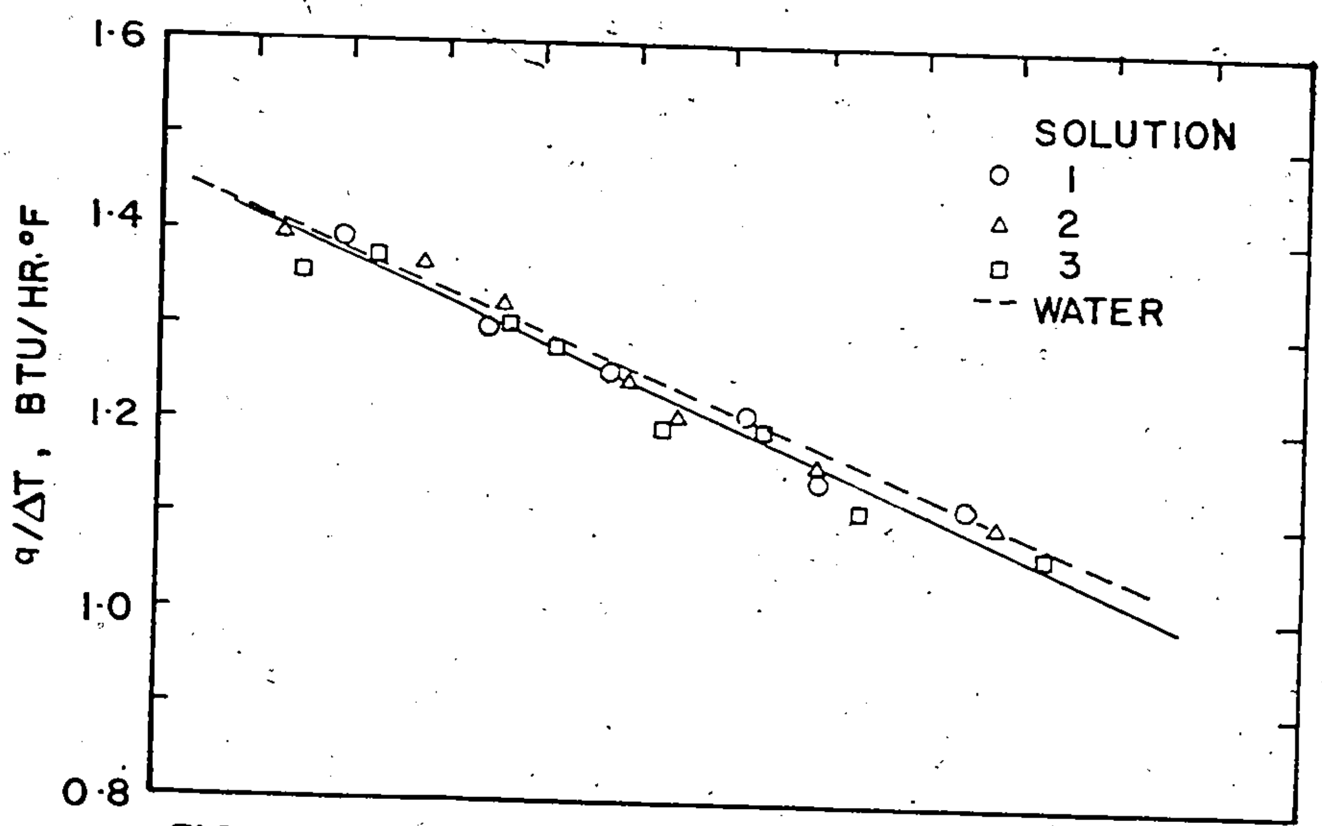


FIG. A3·5 q/ΔT VS T FOR CARBOPOL 934

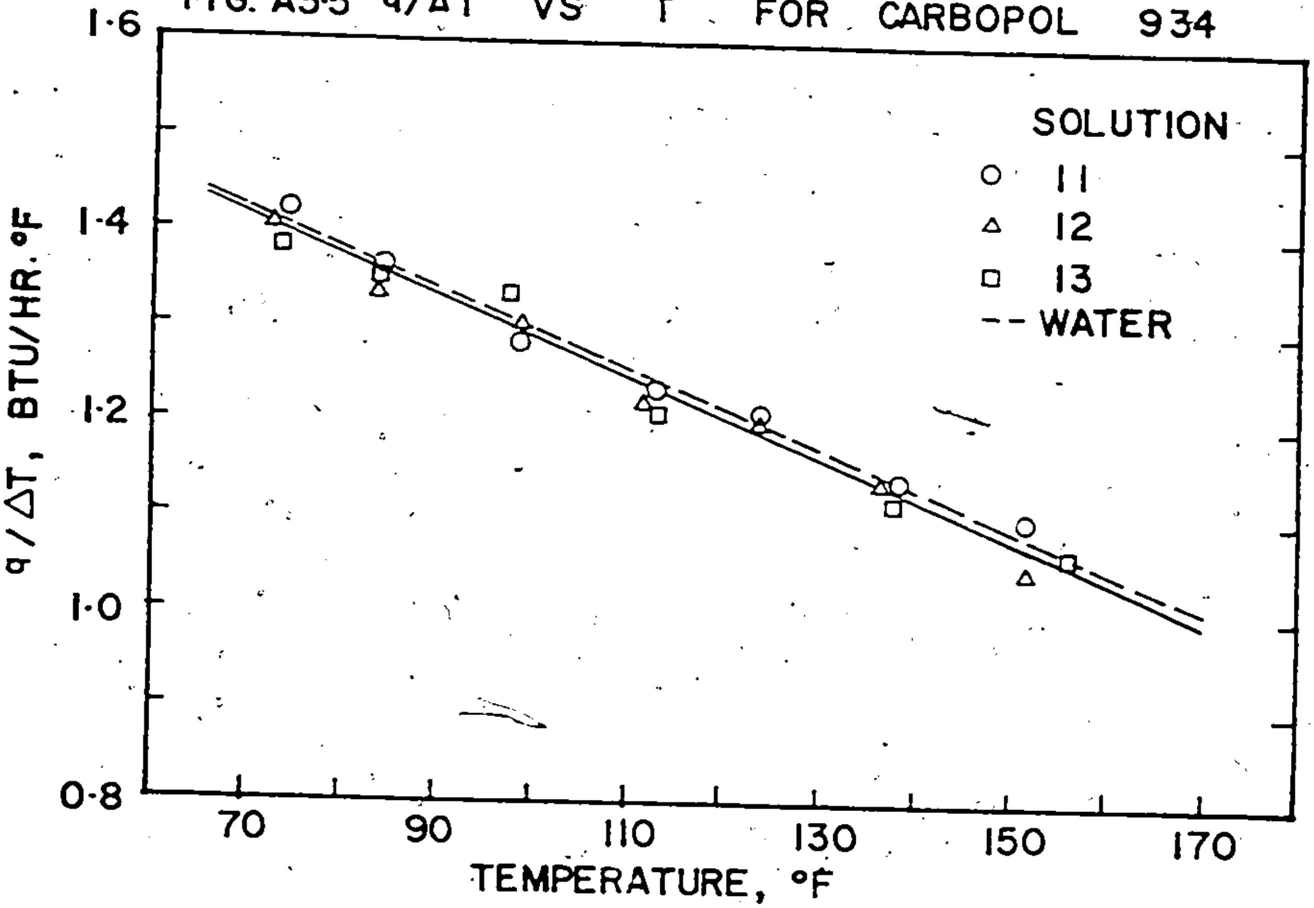


FIG. A3·6 q/ΔT VS T FOR NATROSOL 250 H

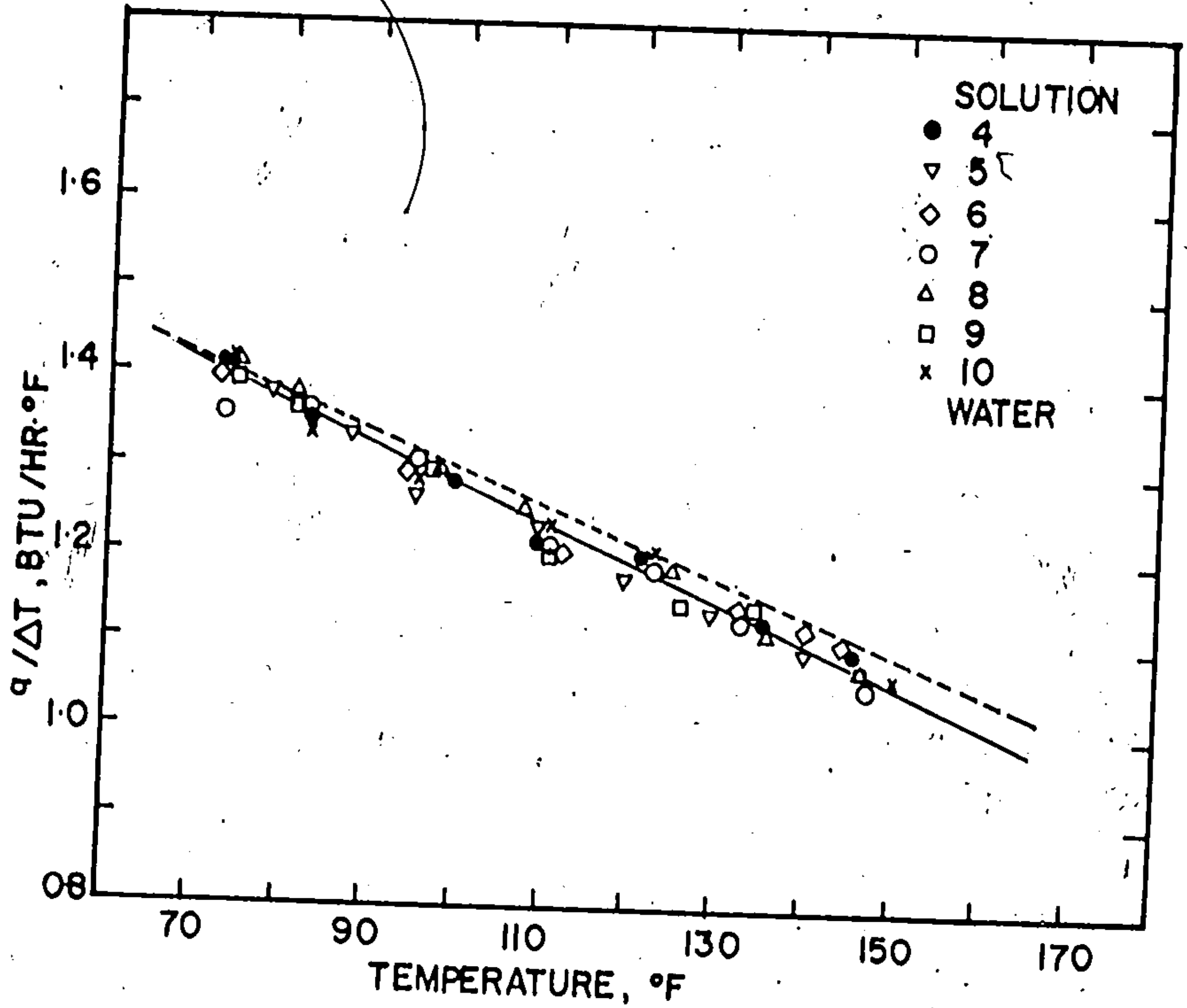


FIG. A3·7 q/ΔT VS T FOR CARBOSE I M.

APPENDIX IV

RHEOLOGICAL PROPERTIES

DIRECTION DETERMINATION OF FLOW CURVE (43)

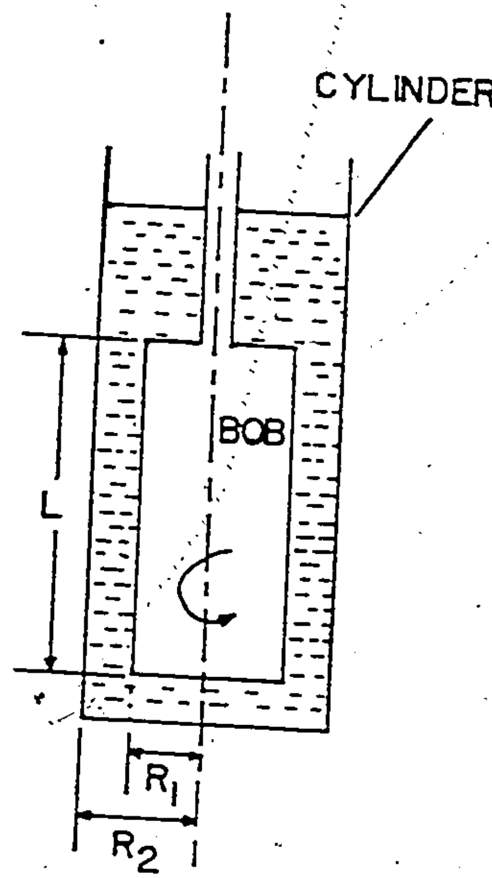


FIG. A4-1 VISCOMETER

Single Bob Method

Symbols :

R_1 = radius of inner cylinder (bob), ft

R_2 = radius of outer cylinder (cup), ft
= 0.04529 ft

$s = R_2/R_1$ = radius ratio

L = actual length of inner cylinder, ft

L_{eq} = equivalent length of inner cylinder, ft

N' = angular velocity, rps

N = angular velocity, rpm

$\Omega = 2\pi N'$

Rd = dial reading of viscometer in %

$M = 4.969 \times 10^{-5}$ Rd, ft-lbf

$\tau = M/(2\pi R_1^2 L_{eq})$

$g(\tau)$ = shearing at bob

ϕ_s = apparent fluidity = reciprocal of apparent viscosity
$= 4\pi R_1^2 R_2^2 L\Omega/M(R_2^2 - R_1^2) = 2\Omega/(\tau(1-1/s^2))$

$g_c = 32.17\ (lb_m/lbf)(ft/sec^2)$

μ = poise

μ' = viscosity, lb_m/ft.sec

For Newtonian fluid,

$$\phi_s = \frac{g_c}{\mu'} = \frac{4\pi R_1^2 R_2^2 L\Omega}{M(R_2^2 - R_1^2)} \qquad \text{(IV.1)}$$

if the bob has no end effect.

But in practical situation, the end effect is usually significant. It is therefore necessary to correct this effect by defining an equivalent length from eqn. (IV.1)

$$\phi_s = \frac{4\pi R_1^2 R_2^2 L_{eq}\Omega}{M(R_2^2 - R_1^2)} \quad \text{or}$$

$$L_{eq} = \frac{[1 - (R_1/R_2)^2]\, g_c M}{8\pi R_1^2 N' \mu'} \tag{IV.2}$$

For Spindle no. 0

R_1 = 0.4953 inch, $(R_1/R_2)^2$ = 0.82955

$$L_{eq} = \frac{[1 - 0.82955] \times 32.17 \times 4.969\times10^{-5}\ Rd}{4 \times 3.1416 \times (0.04125)^2 \times 2 \times 3.1416\ N'\mu}$$

$$= \frac{2.028\times10^{-3}\ Rd}{N'}$$

$$= 21.6637 \frac{Rd}{N\mu} \tag{IV.3}$$

For Spindle no. 1,

R_1 = 0.3078 inch

$$L_{eq} = 121.5 \frac{Rd}{N\mu} \tag{IV.4}$$

For Spindle no. 2

R_1 = 0.3682 inch

$$L_{eq} = 124.6 \frac{Rd}{N\mu} \tag{IV.5}$$

Equations (IV.3) to (IV.5) are used for the calibration of spindles with Newtonian fluids.

For non-Newtonian fluids, the rheological properties were evaluated as follows:

(1). $\tau = \frac{4.969 \times 10^{-5}\ Rd}{2\ R_1^2\ L_{eq}}$ (IV.6)

(2). $s = (R_2/R_1) > 1$ (IV.7)

(3). $k_1 = \frac{s^2-1}{2s^2}\ (1 + 2/3\ \ln s)$ (IV.8)

(4). $k_2 = \frac{s^2-1}{6s^2}\ \ln s$ (IV.9)

(5). $\phi_s = 2\Omega/[\tau(1 - \frac{1}{s^2})]$ (IV.10)

(6). The Least Square Method was used to obtain m in eqn. (IV.11)

$$\frac{d\ \log \phi s}{d\ \log F_1} = m - 1 \qquad \text{(IV.11)}$$

(7). $g(\tau) = \tau\phi_s\ [1 + k_1(m-1) + k_2(m-1)^2]$ (IV.12)

The consistancy index K and flow behaviour index n in the power-law model

$$\tau_{yx} = K\ \left(\frac{\partial u}{\partial y}\right)^{n-1}\ \left(\frac{\partial u}{\partial y}\right) \qquad (3.6)$$

could be evaluated from the shearing stress and the shearing rate obtained with the Brookfield viscometer.

TABLE A4.1

EQUIVALENT LENGTH OF SPINDLE NO. 0

SOLUTION	T °F	SPEED RPM	DIAL READING % .	EQUIVALENT LENGTH INCH.
WATER	68	12	2.52	4.540
		30	5.81	4.105
		60	11.65	3.975
ETHYLENE GLYCOL	68	0.3	1.33	4.826
		0.5	2.33	4.228
		1.5	5.70	4.137
		3	11.04	4.006
		6	21.53	3.906
		12	42.72	3.876
	39.95	1.5	2.82	4.446
		3	5.35	4.218
		6	10.32	4.068
		12	20.13	3.967
		30	49.31	3.887
		60	99.38	3.917
S200-68203	99.9	0.5	18.63	3.911
		1.5	45.27	3.801
		3	90.05	3.781
	68	0.3	2 .45	3.842
		0.6	58.03	3.784
S-6-57306	68	0.3	0.62	4.715
		0.5	1.08	4.106
		1.5	3.0	4.563
		3	5.8	4.416
		6	10.78	4.099
		12	20.48	3.893
		30	49.5	3.764
		60	98.9	3.760

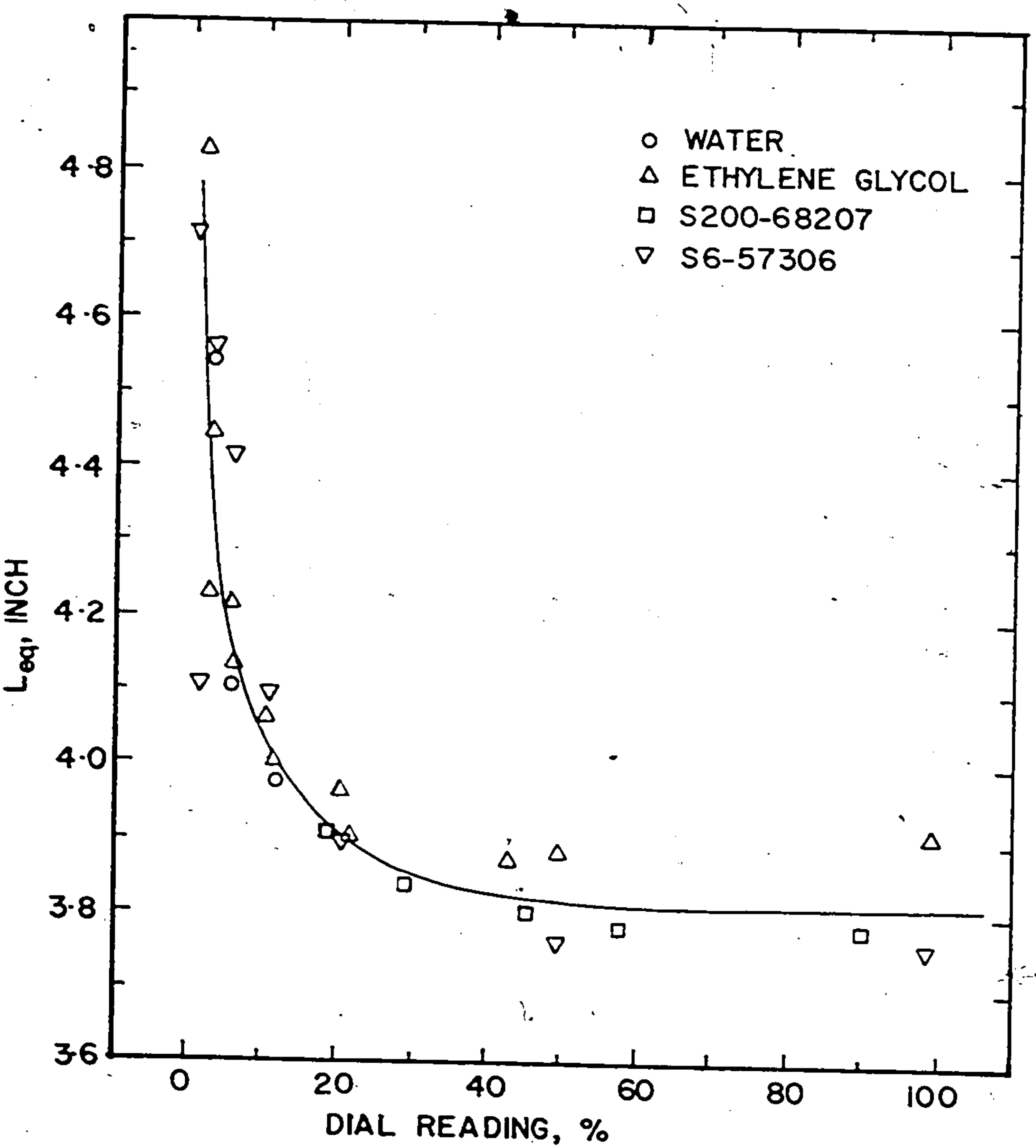


FIG. A4·2 EQUIVALENT LENGTH OF SPINDLE NO. O

TABLE A4.2

EQUIVALENT LENGTH OF SPINDLE NO. 1

SOLUTION	T °F	SPEED RPM	DIAL READING %	EQUIVALENT LENGTH INCH.
S-200-68203	68	0.3	3.95	2.890
		0.6	7.80	2.853
		1.5	19.53	2.858
		3	39.04	2.856
		6	77.82	2.846
	77	3	27.52	2.873
		6	54.87	2.864
	80.6	6	48.1	2.863
		12	95.1	2.834
S-20-68207	68	1.5	1.45	2.869
		3	2.9	2.869
		6	5.61	2.775
		12	11.29	2.792
		30	28.48	2.817
		60	57.01	2.820

TABLE A4.3

EQUIVALENT LENGTH OF SPINDLE NO. 2

SOLUTION	T °F	SPEED RPM	DIAL READING %	EQUIVALENT LENGTH INCH.
S-200-68203	68	0.6	1.33	0.499
		1.5	3.53	0.530
		3	6.8	0.510
		6	13.52	0.507
		12	27.28	0.512
		30	67.71	0.508
	77	1.5	2.43	0.520
		3	4.67	0.500
		6	9.48	0.507
		12	19.0	0.509
		30	47.44	0.529
		60	98.75	0.520
S-20-68207	68	12	2.05	0.520
		30	4.89	0.496
		60	10.03	0.519

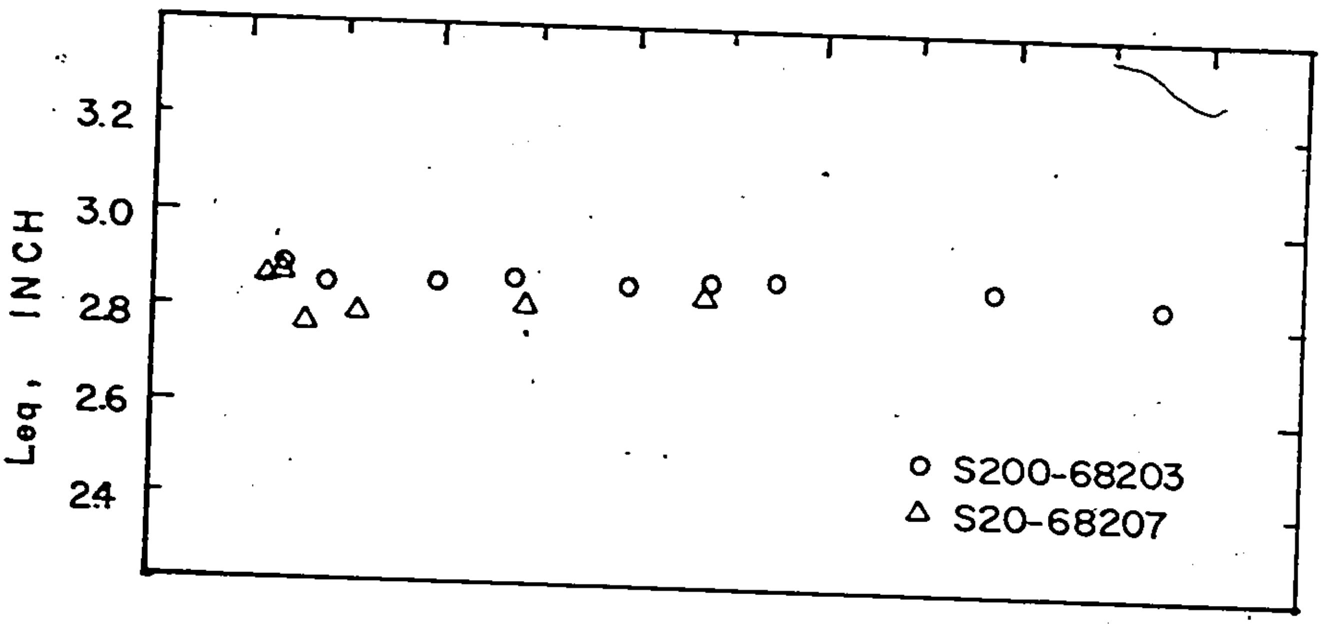


FIG. A4.3 EQUIVALENT LENGTH OF SPINDLE NO. 1

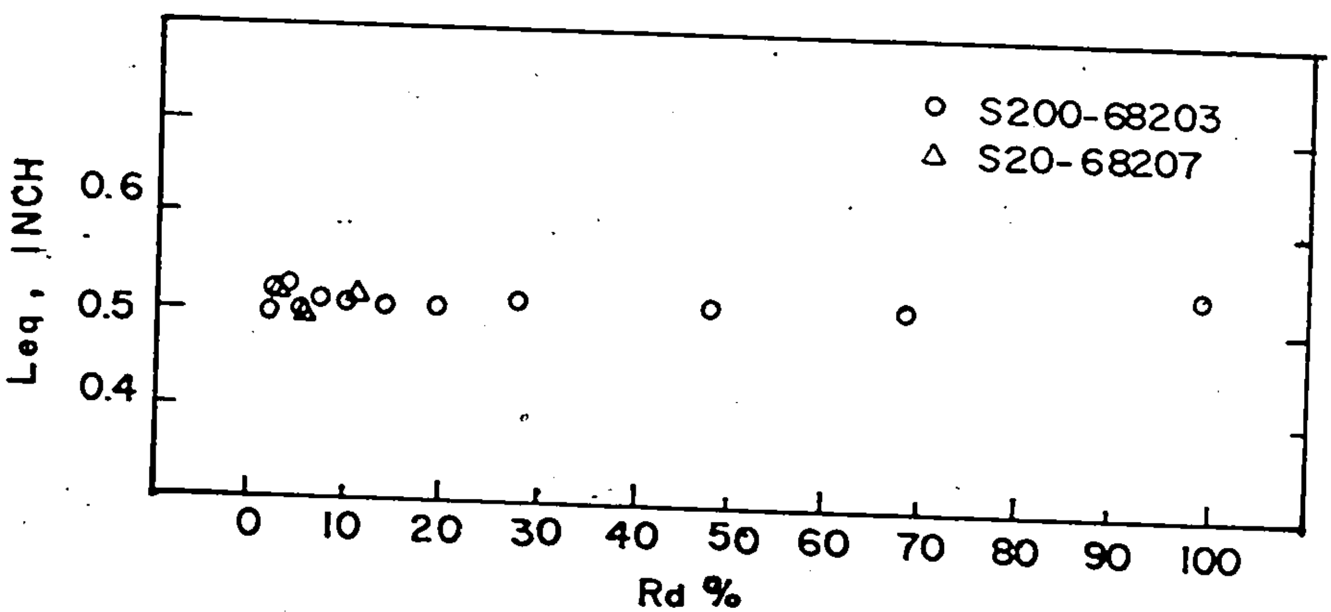


FIG. A4.4 EQUIVALENT LENGTH OF SPINDLE NO. 2

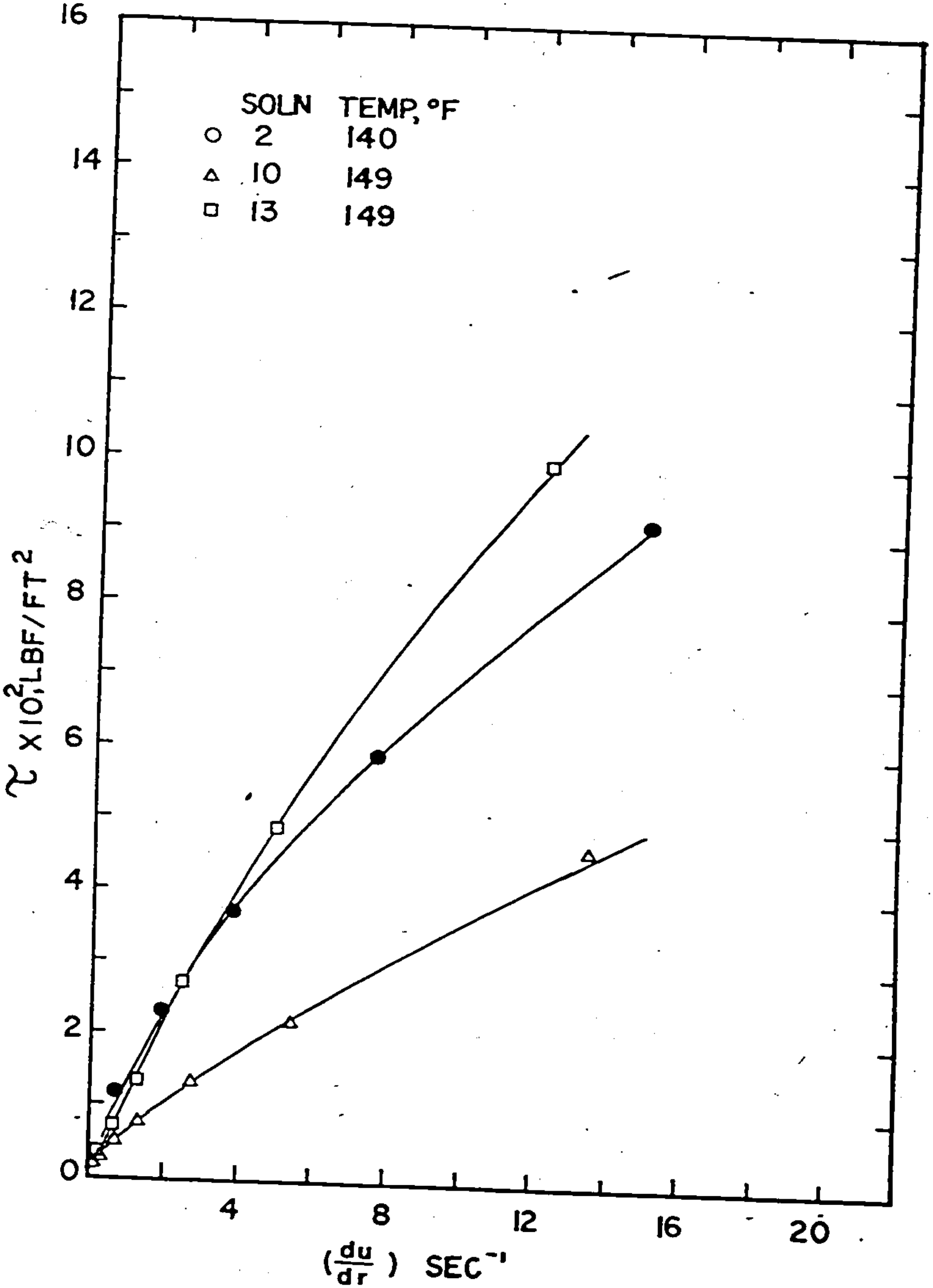


FIG. A4.5 FLOW CURVE OF POLYMER SOLNS.

TABLE A4.4 FLOW CURVE DATA -- SOLUTION 1

TEMP. °F	SPEED RPM	DIAL RD., %	STRESS LBF/FT2	RATE SEC^{-1}	LEQ. INCH
	60.0	49.75	0.7256553E-02	0.7316672E 02	3.820
	30.0	26.07	0.3753452E-02	0.3658336E 02	3.870
68.0	12.0	11.45	0.1588987E-02	0.1463334E 02	4.015
	6.0	6.38	0.8590233E-03	0.7316669E 01	4.135
	3.0	3.40	0.4421081E-03	0.3658336E 01	4.285
	60.0	46.28	0.6750416E-02	0.7313698E 02	3.820
	30.0	24.03	0.3450822E-02	0.3656848E 02	3.880
77.0	12.0	10.79	0.1491821E-02	0.1462739E 02	4.030
	6.0	5.65	0.7477675E-03	0.7313696E 01	4.210
	3.0	3.22	0.4172418E-03	0.3656849E 01	4.300
	60.0	43.77	0.6375961E-02	0.7303838E 02	3.825
	30.0	22.64	0.3241188E-02	0.3651918E 02	3.892
86.0	12.0	9.95	0.1368890E-02	0.1460767E 02	4.050
	6.0	5.36	0.7161908E-03	0.7303837E 01	4.170
	3.0	2.90	0.3706056E-03	0.3651920E 01	4.360
	60.0	40.98	0.5961750E-02	0.7304095E 02	3.830
	30.0	21.53	0.3075955E-02	0.3652048E 02	3.900
95.0	12.0	9.32	0.1282216E-02	0.1460818E 02	4.050
	6.0	5.03	0.6696885E-03	0.7304092E 01	4.185
	3.0	2.75	0.3498315E-03	0.3652046E 01	4.380
	60.0	38.63	0.5619872E-02	0.7303539E 02	3.830
	30.0	20.20	0.2878559E-02	0.3651770E 02	3.910
104.0	12.0	8.68	0.1185387E-02	0.1460708E 02	4.080
	6.0	4.80	0.6360274E-03	0.7303537E 01	4.205
	3.0	2.58	0.3267140E-03	0.3651771E 01	4.400
	60.0	36.62	0.5320512E-02	0.7304607E 02	3.835
	30.0	19.39	0.2757490E-02	0.3652304E 02	3.918
113.0	12.0	8.52	0.1163536E-02	0.1460921E 02	4.080
	6.0	4.60	0.6095262E-03	0.7304605E 01	4.205
	3.0	2.45	0.3102515E-03	0.3652305E 01	4.400
	60.0	34.85	0.5056757E-02	0.7287297E 02	3.840
	30.0	18.30	0.2597837E-02	0.3643648E 02	3.925
122.0	12.0	8.29	0.1130740E-02	0.1457459E 02	4.085
	6.0	4.20	0.5558629E-03	0.7287293E 01	4.210
	3.0	2.16	0.2735278E-03	0.3643649E 01	4.400
	60.0	33.00	0.4782092E-02	0.7294797E 02	3.845
	30.0	17.07	0.2415230E-02	0.3647397E 02	3.938
131.0	12.0	7.51	0.1019359E-02	0.1458959E 02	4.105
	6.0	4.10	0.5400626E-03	0.7294794E 01	4.230

TABLE A4.4 FLOW CURVE DATA -- SOLATION 1 (CONTD.)

TEMP. °F	SPEED RPM	DIAL RD., %	STRESS LBF/FT^2	RATE SEC^{-1}	LEQ. INCH
	3.0	2.10	0.2635345E-03	0.3647398E 01	4.440
	60.0	31.30	0.4529852E-02	0.7288591E 02	3.850
	30.0	16.33	0.2306427E-02	0.3644296E 02	3.945
140.0	12.0	7.17	0.9701371E-03	0.1457718E 02	4.118
	6.0	3.80	0.4958566E-03	0.7288590E 01	4.270
	3.0	1.98	0.2462566E-03	0.3644297E 01	4.480
	60.0	29.89	0.4323546E-02	0.7290866E 02	3.852
	30.0	15.65	0.2205353E-02	0.3645433E 02	3.954
149.0	12.0	6.80	0.9185129E-03	0.1458173E 02	4.125
	6.0	3.50	0.4556431E-03	0.7290863E 01	4.280
	3.0	1.96	0.2432263E-03	0.3645432E 01	4.490

TABLE A4.5 FLOW CURVE DATA -- SOLUTION 2

TEMP. °F	SPEED RPM	DIAL RD., %	STRESS LBF/FT^2	RATE SEC^{-1}	LEQ. INCH
	30.0	98.27	0.1433370E-01	0.3705103E 02	3.820
	12.0	47.25	0.5891903E-02	0.1482041E 02	3.820
68.0	6.0	27.23	0.3925532E-02	0.7410207E 01	3.865
	3.0	15.60	0.2197752E-02	0.3705103E 01	3.955
	1.5	9.05	0.1240476E-02	0.1852551E 01	4.065
	0.6	4.33	0.5723885E-03	0.7410209E 00	4.215
	12.0	51.86	0.7564317E-02	0.1488026E 02	3.820
	6.0	30.33	0.4383776E-02	0.7440128E 01	3.855
77.0	3.0	17.72	0.2512301E-02	0.3720066E 01	3.930
	1.5	10.45	0.1444096E-02	0.1860033E 01	4.032
	0.6	5.25	0.7014929E-03	0.7440132E 00	4.170
	12.0	54.91	0.8009192E-02	0.1493837E 02	3.820
	6.0	32.69	0.4735719E-02	0.7469187E 01	3.845
86.0	3.0	19.57	0.2785222E-02	0.3734594E 01	3.915
	1.5	11.67	0.1621537E-02	0.1867297E 01	4.010
	0.6	5.98	0.8028853E-03	0.7469189E 00	4.150
	12.0	56.87	0.8295078E-02	0.1498637E 02	3.820
	6.0	34.43	0.4995812E-02	0.7493186E 01	3.840
95.0	3.0	20.83	0.2972137E-02	0.3746594E 01	3.905
	1.5	12.53	0.1747571E-02	0.1873297E 01	3.995
	0.6	6.57	0.3863716E-03	0.7493188E 00	4.130

TABLE A4.5 FLOW CURVE DATA -- SOLUTION 2 (CONTD.)

TEMP. °F	SPEED RPM	DIAL RD., %	STRESS LBF/FT^2	RATE SEC^{-1}	LEQ. INCH
	12.0	58.73	0.3566376E-02	0.1502517E 02	3.820
	6.0	35.95	0.5216368E-02	0.7512581E 01	3.840
104.0	3.0	22.14	0.3167165E-02	0.3756293E 01	3.895
	1.5	13.49	0.1888554E-02	0.1878146E 01	3.980
	0.6	7.05	0.9545963E-03	0.7512586E 00	4.115
	12.0	59.93	0.3741409E-02	0.1507957E 02	3.820
	6.0	37.20	0.5404782E-02	0.7539782E 01	3.835
113.0	3.0	23.24	0.3332223E-02	0.3769893E 01	3.886
	1.5	14.30	0.2006994E-02	0.1884946E 01	3.970
	0.6	7.64	0.1038269E-02	0.7539784E 00	4.100
	12.0	61.13	0.8916445E-02	0.1513254E 02	3.820
	6.0	38.22	0.5557325E-02	0.7566269E 01	3.832
122.0	3.0	24.03	0.3449044E-02	0.3783136E 01	3.882
	1.5	15.20	0.2140858E-02	0.1891568E 01	3.956
	0.6	8.18	0.1115736E-02	0.7566271E 00	4.085
	12.0	62.85	0.9167325E-02	0.1516280E 02	3.820
	6.0	39.90	0.5804628E-02	0.7581396E 01	3.830
131.0	3.0	25.49	0.3666156E-02	0.3790700E 01	3.874
	1.5	16.01	0.2258367E-02	0.1895349E 01	3.950
	0.6	8.75	0.1187669E-02	0.7581400E 00	4.105
	12.0	63.15	0.9211078E-02	0.1522974E 02	3.820
	6.0	40.55	0.5899191E-02	0.7614870E 01	3.830
140.0	3.0	25.93	0.3731364E-02	0.3807435E 01	3.872
	1.5	16.47	0.2326201E-02	0.1903718E 01	3.945
	0.6	9.33	0.1282009E-02	0.7614869E 00	4.055
	12.0	64.13	0.9354025E-02	0.1528568E 02	3.820
	6.0	41.40	0.6022848E-02	0.7642838E 01	3.830
149.0	3.0	26.86	0.3869191E-02	0.3821420E 01	3.868
	1.5	17.47	0.2473709E-02	0.1910709E 01	3.935
	0.6	9.89	0.1360635E-02	0.7642840E 00	4.050

TABLE A4.6 FLOW CURVE DATA -- SOLUTION 3

TEMP. °F	SPEED RPM	DIAL RD., %	STRESS LBF/FT^2	RATE SEC^{-1}	LEQ. INCH
77.0	60.0	95.34	0.3330543E-01	0.3156947E 02	2.845
	30.0	63.31	0.2211628E-01	0.1578474E 02	2.845
	12.0	38.19	0.1334104E-01	0.6313895E 01	2.845
	6.0	26.33	0.9197943E-02	0.3156946E 01	2.845
	3.0	18.72	0.6539516E-02	0.1578473E 01	2.845
	1.5	13.19	0.4607704E-02	0.7892370E 00	2.845
	0.6	8.50	0.2969330E-02	0.3156947E 00	2.845
	0.3	6.20	0.2165864E-02	0.1578473E 00	2.845
95.0	30.0	79.52	0.2777897E-01	0.1796667E 02	2.845
	12.0	50.71	0.1771468E-01	0.7186670E 01	2.845
	6.0	36.90	0.1289038E-01	0.3593337E 01	2.845
	3.0	27.45	0.9589195E-02	0.1796668E 01	2.845
	1.5	20.98	0.7329006E-02	0.8983347E 00	2.845
	0.6	14.75	0.5152661E-02	0.3593336E 00	2.845
	0.3	11.47	0.4006848E-02	0.1796668E 00	2.845
113.0	30.0	94.40	0.3297703E-01	0.1985568E 02	2.845
	12.0	62.45	0.2181585E-01	0.7942275E 01	2.845
	6.0	47.42	0.1656538E-01	0.3971137E 01	2.845
	3.0	36.52	0.1275764E-01	0.1985568E 01	2.845
	1.5	28.93	0.1010620E-01	0.9927849E 00	2.845
	0.6	21.60	0.7545598E-02	0.3971139E 00	2.845
	0.3	17.60	0.5148264E-02	0.1985568E 00	2.845
131.0	12.0	72.73	0.2540700E-01	0.9117892E 01	2.845
	6.0	56.42	0.1970937E-01	0.4558945E 01	2.845
	3.0	44.77	0.1563965E-01	0.2279472E 01	2.845
	1.5	36.20	0.1264585E-01	0.1139736E 01	2.845
	0.6	28.05	0.9798795E-02	0.4558946E 00	2.845
	0.3	23.66	0.8265223E-02	0.2279473E 00	2.845
149.0	12.0	81.33	0.2841124E-01	0.9695717E 01	2.845
	6.0	64.13	0.2240273E-01	0.4847856E 01	2.845
	3.0	51.55	0.1800812E-01	0.2423928E 01	2.845
	1.5	42.52	0.1485365E-01	0.1211965E 01	2.845
	0.6	33C47	0.1169218E-01	0.4847855E 00	2.845
	0.3	28C60	0.9990931E-02	0.2423930E 00	2.845

TABLE A4.7 FLOW CURVE DATA -- SOLUTION 4

TEMP. ^{o}F	SPEED RPM	DIAL RD.. %	STRESS LBF/FT^2	RATE SEC^{-1}	LEQ. INCH
	60.0	36.81	0.5348116E-02	0.7288219E 02	3.835
	30.0	19.04	0.2713255E-02	0.3644110E 02	3.910
68.2	12.0	8.49	0.1162288E-02	0.1457643E 02	4.070
	6.0	4.23	0.5585067E-03	0.7288218E 01	4.220
	3.0	2.35	0.2969136E-03	0.3644109E 01	4.410
	60.0	35.53	0.5155426E-02	0.7273369E 02	3.840
	30.0	17.97	0.2547746E-02	0.3636682E 02	3.930
70.5	12.0	7.33	0.9937165E-03	0.1454673E 02	4.110
	6.0	3.96	0.5179478E-03	0.7273365E 01	4.260
	3.0	2.13	0.2666984E-03	0.3636683E 01	4.450
	60.0	26.47	0.3811040E-02	0.7269849E 02	3.870
	30.0	14.13	0.1983135E-02	0.3634924E 02	3.970
88.0	12.0	5.64	0.7554155E-03	0.1453969E 02	4.160
	6.0	2.93	0.3753002E-03	0.7269845E 01	4.350
	3.0	1.64	0.2008321E-03	0.3634925E 01	4.550
	60.0	23.55	0.3373199E-02	0.7271461E 02	3.890
	30.0	11.95	0.1660443E-02	0.3635733E 02	4.010
101.7	12.0	4.67	0.6180666E-03	0.1454292E 02	4.210
	6.0	2.60	0.3299967E-03	0.7271461E 01	4.390
	3.0	1.44	0.1759536E-03	0.3635731E 01	4.560
	60.0	20.83	0.2975948E-02	0.7269916E 02	3.900
	30.0	10.33	0.1424687E-02	0.3634958E 02	4.040
119.0	12.0	4.15	0.5479439E-03	0.1453983E 02	4.220
	6.0	2.24	0.2823749E-03	0.7269916E 01	4.420
	3.0	1.27	0.1538321E-03	0.3634959E 01	4.600
	60.0	17.44	0.2469461E-02	0.7250743E 02	3.935
127.0	30.0	8.[illegible]6	0.1226631E-02	0.3625371E 02	4.070
	12.0	3.60	0.4686615E-03	0.1450149E 02	4.280
	6.0	1.90	0.2357806E-03	0.7250742E 01	4.490
	60.0	15.71	0.[illegible]6050E-02	0.7286079E 02	3.950
138.4	30.0	8.13	0.1110275E-02	0.3643037E 02	4.080
	12.0	3.44	0.4457489E-03	0.1457215E 02	4.300
	6.0	1.93	0.2389713E-03	0.7286077E 01	4.500
	60.0	13.83	0.1936153E-02	0.7317082E 02	3.980
154.0	30.0	7.11	0.9638916E-03	0.3658540E 02	4.110
	12.0	2.96	0.3782734E-03	0.1463416E 02	4.360
	6.0	1.93	0.2395035E-03	0.7317080E 01	4.490
	60.0	12.87	0.1797240E-02	0.7316356E 02	3.990
165.0	30.0	6.91	0.9345042E-03	0.3658176E 02	4.120
	12.0	2.97	0.3804239E-03	0.1463271E 02	4.350
	6.0	1.75	0.2152487E-03	0.7316355E 01	4.530

TABLE A4.8 FLOW CURVE DATA -- SOLUTION 5

TEMP. °F	SPEED RPM	DIAL RD., %	STRESS LBF/FT^2	RATE SEC^{-1}	LEQ. INCH
	60.0	87.35	0.1274090E-01	0.7327180E 02	3.820
	30.0	45.00	0.6555136E-02	0.3663591E 02	3.825
	12.0	18.77	0.2664558E-02	0.1465436E 02	3.925
67.6	6.0	10.31	0.1425457E-02	0.7327185E 01	4.030
	3.0	5.87	0.7862216E-03	0.3663591E 01	4.160
	1.5	3.37	0.4366785E-03	0.1831796E 01	4.300
	0.6	1.48	0.1804456E-03	0.7327187E 00	4.570
	60.0	66.15	0.9648662E-02	0.7326971E 02	3.820
	30.0	33.90	0.4918911E-02	0.3663484E 02	3.840
	12.0	14.26	0.2002389E-02	0.1465394E 02	3.968
84.7	6.0	8.02	0.1095253E-02	0.7326966E 01	4.080
	3.0	4.70	0.6235181E-03	0.3663484E 01	4.200
	1.5	2.52	0.3191161E-03	0.1831743E 01	4.400
	60.0	49.46	0.7214252E-02	0.7344942E 02	3.820
	30.0	25.37	0.3647953E-02	0.3672469E 02	3.875
104.7	12.0	10.83	0.1497352E-02	0.1468987E 02	4.030
	6.0	6.43	0.8674834E-03	0.7344935E 01	4.130
	3.0	3.60	0.4675691E-03	0.3672470E 01	4.290
	1.5	2.10	0.2635345E-03	0.1836234E 01	4.440
	60.0	39.47	0.5742073E-02	0.7348428E 02	3.830
	30.0	20.22	0.2885099E-02	0.3674214E 02	3.905
121.5	12.0	8.86	0.1215928E-02	0.1469685E 02	4.060
	6.0	5.20	0.5948125E-03	0.7348423E 01	4.170
	3.0	3.00	0.3851519E-03	0.3674213E 01	4.340
	1.5	1.67	0.2091016E-03	0.1837107E 01	4.450
	60.0	32.73	0.4742965E-02	0.7359875E 02	3.845
	30.0	16.65	0.2354607E-02	0.3679938E 02	3.940
136.4	12.0	7.57	0.1028756E-02	0.1471974E 02	4.100
	6.0	4.49	0.5942441E-03	0.7359872E 01	4.210
	3.0	2.65	0.3363427E-03	0.3679937E 01	4.390
	1.5	1.45	0.1771755E-03	0.1839969E 01	4.560
	60.0	28.30	0.4085071E-02	0.7366624E 02	3.860
	30.0	14.78	0.2079600E-02	0.3683311E 02	3.960
147.7	12.0	6.50	0.8769277E-03	0.1473325E 02	4.130
	6.0	3.90	0.5113005E-03	0.7366623E 01	4.250
	3.0	2.27	0.2894308E-03	0.3683312E 01	4.370
	1.5	1.33	0.1618032E-03	0.1841657E 01	4.580
	60.0	24.37	0.3499647E-02	0.7364886E 02	3.880
	30.0	12.67	0.1769311E-02	0.3682443E 02	3.990
159.4	12.0	6.06	0.8155911E-03	0.1472977E 02	4.140
	6.0	3.50	0.4535238E-03	0.7364883E 01	4.300
	3.0	2.02	0.2517935E-03	0.3682443E 01	4.470
	60.0	22.50	0.3222801E-02	0.7358121E 02	3.890
	30.0	11.55	0.1604863E-02	0.3679060E 02	4.010
166.8	12.0	4.78	0.6522402E-03	0.1471624E 02	4.190
	6.0	3.03	0.3909044E-03	0.7358119E 01	4.320
	3.0	1.73	0.2118533E-03	0.3679061E 01	4.550
	1.5	1.05	0.1258163E-03	0.1839531E 01	4.650

TABLE A4.9 FLOW CURVE DATA -- SOLUTION 6

TEMP. °F	SPEED RPM	DIAL RD., %	STRESS LBF/FT2	RATE SEC -1	LEQ. INCH
	12.0	80.27	0.1170821E-01	0.1467086E 02	3.820
	6.0	42.68	0.5209064E-02	0.7335430E 01	3.830
67.6	3.0	22.81	0.3267203E-02	0.3667717E 01	3.890
	1.5	12.23	0.1703598E-02	0.1833858E 01	4.000
	0.6	5.42	0.7242083E-03	0.7335433E 00	4.170
	0.3	3.33	0.4314955E-03	0.3667717E 00	4.300
	12.0	61.66	0.3993749E-02	0.1464977E 02	3.820
	6.0	32.78	0.4744042E-02	0.7324883E 01	3.850
81.0	3.0	17.35	0.2453602E-02	0.3662443E 01	3.940
	1.5	9.36	0.1284547E-02	0.1831221E 01	4.060
	0.6	4.34	0.5730307E-03	0.7324886E 00	4.220
	0.3	2.39	0.3012840E-03	0.3662443E 00	4.420
	12.0	48.26	0.7039219E-02	0.1463053E 02	3.820
	6.0	25.68	0.3697299E-02	0.7315267E 01	3.870
94.1	3.0	13.79	0.1887863E-02	0.3657635E 01	4.070
	1.5	7.51	0.1020602E-02	0.1828815E 01	4.100
	0.6	3.51	0.1548195E-03	0.7315268E 00	4.300
	0.3	1.78	0.2179762E-03	0.3657634E 00	4.550
	12.0	38.30	0.5571865E-02	0.1465006E 02	3.830
	6.0	20.22	0.2885099E-02	0.7325031E 01	3.905
106.9	3.0	10.98	0.1521867E-02	0.3662516E 01	4.020
	1.5	5.96	0.8001993E-03	0.1831257E 01	4.150
	0.6	2.83	0.3608323E-03	0.7325032E 00	4.370
	0.3	1.50	0.1828841E-03	0.3662517E 00	4.570
	12.0	28.52	0.4116829E-02	0.1467589E 02	3.860
	6.0	15.28	0.2152669E-02	0.7337951E 01	3.955
125.2	3.0	8.49	0.1159439E-02	0.3668974E 01	4.080
	1.5	4.70	0.6220371E-03	0.1834486E 01	4.210
	0.6	2.18	0.2729590E-03	0.7337950E 00	4.450
	30.0	46.04	0.6715409E-02	0.3668307E 02	3.820
	12.0	19.77	0.2817283E-02	0.1467323E 02	3.910
147.7	6.0	10.55	0.1456832E-02	0.7336618E 01	4.035
	3.0	5.82	0.7795249E-03	0.3668307E 01	4.160
	1.5	3.41	0.4428918E-03	0.1834153E 01	4.290
	0.6	1.53	0.1915721E-03	0.7336619E 00	4.450

TABLE A4.10 FLOW CURVE DATA -- SOLUTION 7

TEMP. °F	SPEED RPM	DIAL RD., %	STRESS LBF/FT ^{2}	RATE SEC $^{-1}$	LEQ. INCH
	60.0	91.68	0.1337246E-01	0.7301534E 02	3.820
	30.0	47.80	0.6972123E-02	0.3650768E 02	3.820
68.0	12.0	20.25	0.2885684E-02	0.1460308E 02	3.910
	6.0	10.65	0.1472465E-02	0.7301536E 01	4.030
	3.0	5.72	0.7661306E-03	0.3650768E 01	4.160
	1.5	3.18	0.4082611E-03	0.1825384E 01	4.340
	60.0	78.33	0.1142523E-01	0.7306424E 02	3.820
	30.0	40.75	0.5928289E-02	0.3653210E 02	3.830
77.0	12.0	17.23	0.2436629E-02	0.1461284E 02	3.940
	6.0	9.27	0.1273765E-02	0.7306418E 01	4.055
	3.0	5.02	0.6691569E-03	0.3653210E 01	4.180
	1.5	2.78	0.3536479E-03	0.1826605E 01	4.380
	60.0	67.53	0.9849951E-02	0.7300031E 02	3.820
	30.0	35.15	0.5100284E-02	0.3650015E 02	3.840
86.0	12.0	14.95	0.2103519E-02	0.1460006E 02	3.960
	6.0	8.00	0.1089850E-02	0.7300032E 01	4.090
	3.0	4.33	0.5717103E-03	0.3650015E 01	4.220
	60.0	58.90	0.8576587E-02	0.7292091E 02	3.820
	30.0	30.33	0.4378099E-02	0.3646045E 02	3.860
95.0	12.0	12.90	0.1801430E-02	0.1458419E 02	3.990
	6.0	6.97	0.9426188E-03	0.7292089E 01	4.120
	3.0	3.66	0.4753619E-03	0.3646047E 01	4.290
	60.0	51.70	0.7540978E-02	0.7297214E 02	3.820
	30.0	26.87	0.3873635E-02	0.3648605E 02	3.865
104.0	12.0	11.38	0.1579273E-02	0.1459443E 02	4.015
	6.0	6.18	0.8317416E-03	0.7297210E 01	4.140
	3.0	3.33	0.4304943E-03	0.3648607E 01	4.310
	60.0	46.19	0.6737288E-02	0.7295230E 02	3.820
	30.0	23.93	0.3436460E-02	0.3647615E 02	3.880
113.0	12.0	10.03	0.1386744E-02	0.1459046E 02	4.030
	6.0	5.43	0.7264160E-03	0.7295227E 01	4.165
	3.0	3.00	0.3842665E-03	0.3647614E 01	4.350
	60.0	41.05	0.5971931E-02	0.7287656E 02	3.830
	30.0	21.40	0.3057382E-02	0.3643829E 02	3.900
122.0	12.0	8.87	0.1214311E-02	0.1457531E 02	4.070
	6.0	4.77	0.6328046E-03	0.7287655E 01	4.200
	3.0	2.60	0.3307499E-03	0.3643828E 01	4.380
	60.0	36.90	0.5361192E-02	0.7280425E 02	3.835
	30.0	18.87	0.2682170E-02	0.3640215E 02	3.920

TABLE A4.10 FLOW CURVE DATA -- SOLATION 7 (CONTD.)

TEMP. °F	SPEED RPM	DIAL RD.. %	STRESS LBF/FT2	RATE SEC -1	LEQ. INCH
131.0	12.0	7.87	0.1069526E-02	0.1456085E 02	4.100
	6.0	4.25	0.5611475E-03	0.7280427E 01	4.220
	3.0	2.25	0.2336357E-03	0.3640215E 01	4.420
140.0	60.0	33.22	0.4813973E-02	0.7294005E 02	3.645
	30.0	17.23	0.2442629E-02	0.3647003E 02	3.930
	12.0	7.29	0.9382937E-03	0.1458801E 02	4.110
	6.0	4.00	0.5268902E-03	0.7294003E 01	4.230
	3.0	2.15	0.2695088E-03	0.3647002E 01	4.440
149.0	60.0	30.32	0.4388023E-02	0.7301814E 02	3.650
	30.0	15.74	0.2222282E-02	0.3650906E 02	3.950
	12.0	6.53	0.9236848E-03	0.1460363E 02	4.120
	6.0	3.75	0.4893322E-03	0.7301813E 01	4.270
	3.0	2.03	0.2530401E-03	0.3650906E 01	4.470

TABLE A4.11 FLOW CURVE DATA -- SOLUTION 8

TEMP. °F	SPEED RPM	DIAL RD.. %	STRESS LBF/FT 2	RATE SEC-1	LEQ. INCH
68.2	12.0	69.24	0.1009937E-01	0.1466037E 02	3.820
	6.0	36.57	0.5306330E-02	0.7330186E 01	3.840
	3.0	19.42	0.2767407E-02	0.3665094E 01	3.910
	1.5	10.51	0.1449512E-02	0.1832547E 01	4.040
	0.6	5.00	0.6633168E-03	0.7330186E 00	4.200
77.0	12.0	57.84	0.8436561E-02	0.1466741E 02	3.820
	6.0	30.52	0.4416969E-02	0.7333706E 01	3.850
	3.0	16.32	0.2364963E-02	0.3666855E 01	3.845
	1.5	8.86	0.1215679E-02	0.1833426E 01	4.070
	0.6	4.25	0.5611475E-03	0.7333709E 00	4.220
86.0	12.0	48.83	0.7122360E-02	0.1463359E 02	3.820
	6.0	25.71	0.3701619E-02	0.7316795E 01	3.870
	3.0	13.41	0.1874999E-02	0.3658401E 01	3.985
	1.5	7.25	0.9828710E-03	0.1829199E 01	4.110
	0.6	3.44	0.4457489E-03	0.7316800E 00	4.300
95.0	30.0	95.25	0.1389318E-01	0.3659462E 02	3.820
	12.0	41.47	0.6033033E-02	0.1463785E 02	3.830
	6.0	21.90	0.3132831E-02	0.7318923E 01	3.895
	3.0	11.69	0.1624316E-02	0.3659465E 01	4.010
	1.5	6.48	0.3742290E-03	0.1829731E 01	4.130

TABLE A4.11 FLOW CURVE DATA -- SOLATION 8 (CONTD.)

TEMP. °F	SPEED RPM	DIAL RD.. %	STRESS LBF/FT^2	RATE SEC^{-1}	LEQ. INCH
	30.0	82.43	0.1202326E-01	0.3660016E 02	3.820
	12.0	35.50	0.5151071E-02	0.1464007E 02	3.840
104.0	6.0	18.63	0.2648058E-02	0.7320033E 01	3.920
	3.0	10.03	0.1383311E-02	0.3660017E 01	4.040
	1.5	5.70	0.7634519E-03	0.1830008E 01	4.160
	30.0	71.10	0.1037066E-01	0.3656960E 02	3.820
	12.0	30.40	0.4399601E-02	0.1462784E 02	3.850
113.0	6.0	16.21	0.2286581E-02	0.7313923E 01	3.950
	3.0	8.60	0.1175902E-02	0.3656960E 01	4.075
	1.5	4.83	0.6407641E-03	0.1828480E 01	4.200
	30.0	62.33	0.9091478E-02	0.3656364E 02	3.820
	12.0	26.70	0.3844154E-02	0.1462546E 02	3.870
122.0	6.0	14.00	0.1964889E-02	0.7312732E 01	3.970
	3.0	7.51	0.1020602E-02	0.3656365E 01	4.100
	1.5	4.25	0.5611475E-03	0.1828182E 01	4.220
	30.0	54.36	0.7928967E-02	0.3652556E 02	3.820
	12.0	23.23	0.3327362E-02	0.1461024E 02	3.890
131.0	6.0	12.16	0.1693846E-02	0.7305123E 01	4.000
	3.0	6.61	0.8917677E-03	0.3652560E 01	4.130
	1.5	3.60	0.4686615E-03	0.1826280E 01	4.280
	60.0	90.38	0.1318285E-01	0.7311516E 02	3.820
	30.0	47.70	0.6957538E-02	0.3655756E 02	3.820
140.0	12.0	20.35	0.2899935E-02	0.1462303E 02	3.910
	6.0	10.70	0.1479378E-02	0.7311512E 01	4.030
	3.0	5.86	0.7848828E-03	0.3655757E 01	4.160
	1.5	3.28	0.4250165E-03	0.1827879E 01	4.300
	60.0	80.39	0.1172570E-01	0.7313506E 02	3.820
	30.0	42.60	0.6197426E-02	0.3656752E 02	3.830
149.0	12.0	17.88	0.2534986E-02	0.1462701E 02	3.930
	6.0	9.75	0.1341375E-02	0.7313503E 01	4.050
	3.0	5.43	0.7255445E-03	0.3656752E 01	4.170

TABLE A4.12 FLOW CURVE DATA -- SOLUTION 9

TEMP. °F	SPEED RPM	DIAL RD., %	STRESS LBF/FT ^{2}	RATE SEC $^{-1}$	LEQ. INCH
	30.0	89.08	0.3111858E-01	0.1234956E 02	2.845
	12.0	40.80	0.1425279E-01	0.4939825E 01	2.845
68.0	6.0	22.43	0.7835541E-02	0.2469912E 01	2.845
	3.0	12.23	0.4272342E-02	0.1234954E 01	2.845
	1.5	6.73	0.2351011E-02	0.6174781E 00	2.845
	30.0	60.50	0.2113465E-01	0.1254656E 02	2.845
	12.0	27.78	0.9704474E-02	0.5018621E 01	2.845
86.0	6.0	15.48	0.5407676E-02	0.2509308E 01	2.845
	3.0	8.39	0.2930905E-02	0.1254655E 01	2.845
	1.5	4.83	0.1687279E-02	0.6273276E 00	2.845
	0.6	2.42	0.8453862E-03	0.2509310E 00	2.845
	60.0	78.87	0.2755191E-01	0.2485616E 02	2.845
	30.0	43.97	0.1536017E-01	0.1242809E 02	2.845
95.0	12.0	20.11	0.7025089E-02	0.4971234E 01	2.845
	6.0	10.95	0.3825198E-02	0.2485616E 01	2.845
	3.0	6.10	0.2130932E-02	0.1242808E 01	2.845
	1.5	3.50	0.1222666E-02	0.6214045E 00	2.845
	30.0	72.79	0.2542796E-01	0.1232006E 02	2.845
	12.0	33.63	0.1174808E-01	0.4928021E 01	2.845
77.0	6.0	18.43	0.6438207E-02	0.2464009E 01	2.845
	3.0	9.88	0.3451412E-02	0.1232004E 01	2.845
	1.5	5.44	0.1900372E-02	0.6160028E 00	2.845
	30.0	88.63	0.3096139E-01	0.1246799E 02	2.845
	12.0	41.59	0.1452876E-01	0.4987197E 01	2.845
	6.0	23.14	0.8083567E-02	0.2493598E 01	2.845
86.0	3.0	12.60	0.4401594E-02	0.1246799E 01	2.845
	1.5	7.10	0.2480265E-02	0.6233993E 00	2.845
	0.6	3.33	0.1163279E-02	0.2493600E 00	2.845
	60.0	92.74	0.3239714E-01	0.2457704E 02	2.845
	30.0	51.57	0.1801512E-01	0.1228853E 02	2.845
95.0	12.0	23.31	0.8142956E-02	0.4915410E 01	2.845
	6.0	12.47	0.4356183E-02	0.2457704E 01	2.845
	3.0	6.72	0.2347518E-02	0.1228852E 01	2.845
	1.5	3.75	0.1309999E-02	0.6144264E 00	2.845
	60.0	67.55	0.2359746E-01	0.2445363E 02	2.845
	30.0	37.70	0.1316986E-01	0.1222683E 02	2.845
113.0	12.0	16.70	0.5833860E-02	0.4890728E 01	2.845
	6.0	9.00	0.3143998E-02	0.2445363E 01	2.845
	3.0	4.80	0.1676799E-02	0.1222682E 01	2.845
	1.5	2.60	0.9082663E-03	0.6113414E 00	2.845

TABLE A4.12 FLOW CURVE DATA -- SOLATION 9 (CONTD.)

TEMP. °F	SPEED RPM	DIAL RD., %	STRESS LBF/FT 2	RATE SEC-1	LEQ. INCH
	60.0	50.35	0.1758893E-01	0.2484753E 02	2.845
	30.0	28.13	0.9826742E-02	0.1242378E 02	2.845
131.0	12.0	12.73	0.4447009E-02	0.4969508E 01	2.845
	6.0	7.06	0.2466291E-02	0.2484755E 01	2.845
	3.0	3.98	0.1390346E-02	0.1242377E 01	2.845
	60.0	39.80	0.1390345E-01	0.2526657E 02	2.845
	30.0	22.19	0.7748205E-02	0.1263330E 02	2.845
149.0	12.0	10.45	0.3650530E-02	0.5053311E 01	2.845
	6.0	6.05	0.2113465E-02	0.2526655E 01	2.845
	3.0	3.45	0.1205199E-02	0.1263328E 01	2.845

TABLE A4.13 FLOW CURVE DATA -- SOLUTION 10

TEMP. OF.	SPEED RPM	DIAL RD., %	STRESS LBF/FT 2	RATE SEC-1	LEQ. INCH
	30.0	84.10	0.1653291E 00	0.1300608E 02	0.511
	12.0	45.50	0.8944684E-01	0.5202429E 01	0.511
	6.0	26.30	0.5170227E-01	0.2601213E 01	0.511
77.0	3.0	15.33	0.3013671E-01	0.1300607E 01	0.511
	1.5	8.99	0.1767313E-01	0.6503040E 00	0.511
	0.6	4.50	0.9023324E-02	0.2601215E 00	0.511
	0.3	2.87	0.5642030E-02	0.1300607E 00	0.511
	30.0	60.00	0.1179519E 00	0.1317411E 02	0.511
	12.0	32.38	0.6365472E-01	0.5269642E 01	0.511
	6.0	19.67	0.3866857E-01	0.2634821E 01	0.511
95.0	3.0	11.50	0.2260744E-01	0.1317410E 01	0.511
	1.5	6.82	0.1340719E-01	0.6587054E 00	0.511
	0.6	3.62	0.7116430E-02	0.2634821E 00	0.511
	0.3	2.21	0.4344560E-02	0.1317410E 00	0.511
	60.0	65.10	0.1279778E 00	0.2612680E 02	0.511
	30.0	41.57	0.8172101E-01	0.1306341E 02	0.511
	12.0	20.75	0.4079170E-01	0.5225360E 01	0.511
113.(	6.0	11.95	0.2349209E-01	0.2612679E 01	0.511
	3.0	7.11	0.1397730E-01	0.1306340E 01	0.511
	1.5	4.23	0.8315608E-02	0.6531706E 00	0.511
	0.6	2.10	0.4128318E-02	0.2612681E 00	0.511
	0.3	1.45	0.2850504E-02	0.1306340E 00	0.511

TABLE A4.13 FLOW CURVE DATA -- SOLATION 10 (CONTD.)

TEMP. °F	SPEED RPM	DIAL RD., %	STRESS LBF/FT^2	RATE SEC^{-1}	LEQ. INCH
	60.0	47.90	0.9416491E-01	0.2624304E 02	0.511
	30.0	30.02	0.5901529E-01	0.1312153E 02	0.511
	12.0	14.51	0.2852470E-01	0.5248610E 01	0.511
140.0	6.0	8.40	0.1651325E-01	0.2624305E 01	0.511
	3.0	4.98	0.9790007E-02	0.1312152E 01	0.511
	1.5	3.16	0.6212134E-02	0.6560765E 00	0.511
	0.6	1.75	0.3440264E-02	0.2624305E 00	0.511
	0.3	1.00	0.1965865E-02	0.1312152E 00	0.511
	60.0	34.88	0.4856936E-01	0.2702818E 02	0.511
	30.0	23.43	0.4606021E-01	0.1351411E 02	0.511
	12.0	11.09	0.2180143E-01	0.5405643E 01	0.511
149.0	6.0	6.44	0.1266016E-01	0.2702821E 01	0.511
	3.0	3.95	0.7765166E-02	0.1351412E 01	0.511
	1.5	2.52	0.4953980E-02	0.6757056E 00	0.511
	0.6	1.57	0.3086408E-02	0.2702821E 00	0.511
	0.3	1.00	0.1956035E-02	0.1351410E 00	0.511

TABLE A4.14 FLOW CURVE DATA -- SOLUTION 11

TEMP. °F	SPEED RPM	DIAL RD.. %	STRESS LBF/FT 2	RATE SEC-1	LEQ. INCH
	30.0	74.28	0.1083450E-01	0.3640428E 02	3.820
	12.0	31.05	0.4493672E-02	0.1456172E 02	3.850
	6.0	16.05	0.2264011E-02	0.7280857E 01	3.950
68.0	3.0	8.30	0.1133492E-02	0.3640429E 01	4.080
	1.5	4.50	0.5955675E-03	0.1820214E 01	4.210
	0.6	1.90	0.2352566E-03	0.7280860E 00	4.500
	0.3	1.00	0.1198251E-03	0.3640432E 00	4.650
	30.0	60.90	0.8882895E-02	0.3642737E 02	3.820
	12.0	25.38	0.3649393E-02	0.1457095E 02	3.875
77.0	6.0	13.05	0.1822377E-02	0.7285476E 01	3.990
	3.0	7.01	0.9503346E-03	0.3642737E 01	4.110
	1.5	3.90	0.5077166E-03	0.1821368E 01	4.280
	0.6	1.58	0.1922173E-03	0.7285476E 00	4.580
	60.0	97.36	0.1420096E-01	0.7293102E 02	3.820
	30.0	50.32	0.7339694E-02	0.3646552E 02	3.820
	12.0	20.92	0.2984978E-02	0.1458621E 02	3.905
86.0	6.0	10.91	0.1512164E-02	0.7293102E 01	4.020
	3.0	5.95	0.7988571E-03	0.3646550E 01	4.150
	1.5	3.14	0.4049921E-03	0.1823275E 01	4.320
	0.6	1.40	0.1703191E-03	0.7293104E 00	4.580
	60.0	80.61	0.1175780E-01	0.7279955E 02	3.820
	30.0	41.55	0.6044671E-02	0.3639977E 02	3.830
95.0	12.0	17.14	0.2423903E-02	0.1455992E 02	3.940
	6.0	8.90	0.1218417E-02	0.7279956E 01	4.070
	3.0	4.75	0.6286341E-03	0.3639978E 01	4.210
	1.5	2.50	0.3165333E-03	0.1819988E 01	4.400
	0.6	1.08	0.1294111E-03	0.7279958E 00	4.650
	60.0	68.47	0.9987056E-02	0.7282202E 02	3.820
	30.0	35.05	0.5085777E-02	0.3641101E 02	3.840
	12.0	14.47	0.2035982E-02	0.1456441E 02	3.960
104.0	6.0	7.63	0.1034387E-02	0.7282199E 01	4.110
	3.0	3.99	0.5230997E-03	0.3641101E 01	4.250
	1.5	2.18	0.2723469E-03	0.1820550E 01	4.460
	0.6	0.93	0.1111982E-03	0.7282206E 00	4.660
	60.0	57.96	0.8454066E-02	0.7276532E 02	3.820
	30.0	29.70	0.4292719E-02	0.3638266E 02	3.855
113.0	12.0	12.23	0.1703598E-02	0.1455307E 02	4.000
	6.0	6.60	0.8925796E-03	0.7276531E 01	4.120
	3.0	3.45	0.4480870E-03	0.3638268E 01	4.290
	1.5	1.80	0.2204255E-03	0.1819134E 01	4.550

TABLE A4.14 FLOW CURVE DATA -- SOLATION II (CONTD.)

TEMP. °F	SPEED RPM	DIAL RD., %	STRESS LBF/FT²	RATE SEC⁻¹	LEQ. INCH
	60.0	49.17	0.7171951E-02	0.7271288E 02	3.820
	30.0	25.32	0.3636073E-02	0.3635643E 02	3.880
122.0	12.0	10.32	0.1454491E-02	0.1454258E 02	4.030
	6.0	5.52	0.7393430E-03	0.7271225E 01	4.160
	3.0	3.03	0.3908044E-03	0.3635644E 01	4.320
	1.5	1.49	0.1777747E-03	0.1817822E 01	4.670
131.0	60.0	42.25	0.6162599E-02	0.7272037E 02	3.820
	30.0	21.43	0.3061668E-02	0.3636017E 02	3.900
	12.0	8.93	0.1224028E-02	0.1454407E 02	4.065
	6.0	4.75	0.5301510E-03	0.7272035E 01	4.200
	3.0	2.47	0.3127842E-03	0.3636019E 01	4.400
	1.5	1.30	0.1578089E-03	0.1818008E 01	4.590
	60.0	36.27	0.5269662E-02	0.7267160E 02	3.835
	30.0	18.38	0.2609194E-02	0.3633580E 02	3.925
140.0	12.0	7.55	0.1026038E-02	0.1453431E 02	4.100
	6.0	4.07	0.5348464E-03	0.7267156E 01	4.240
	3.0	2.08	0.2586937E-03	0.3633579E 01	4.480
	1.5	1.10	0.1320917E-03	0.1816789E 01	4.640
	60.0	31.20	0.4515380E-02	0.7272864E 02	3.850
	30.0	15.90	0.2242852E-02	0.3636432E 02	3.950
149.0	12.0	6.80	0.9196280E-03	0.1454572E 02	4.120
	6.0	3.68	0.4790763E-03	0.7272860E 01	4.280
	3.0	1.78	0.2179762E-03	0.3636431E 01	4.550
	1.5	1.00	0.1198251E-03	0.1818215E 01	4.650

TABLE A4.15 FLOW CURVE DATA -- SOLUTION 12

TEMP. °F	SPEED RPM	DIAL RD., %	STRESS LBF/FT^2	RATE SEC^{-1}	LEQ. INCH
	6.0	93.84	0.3278142E-01	0.2377916E 01	2.845
	3.0	50.48	0.1763433E-01	0.1188958E 01	2.845
68.0	1.5	26.30	0.9187464E-02	0.5944795E 00	2.845
	0.6	10.60	0.3702931E-02	0.2377917E 00	2.845
	0.3	5.40	0.1886398E-02	0.1188958E 00	2.845
	6.0	72.67	0.2538604E-01	0.2367672E 01	2.845
	3.0	38.52	0.1345631E-01	0.1183836E 01	2.845
77.0	1.5	19.80	0.6916799E-02	0.5919183E 00	2.845
	0.6	8.01	0.2798158E-02	0.2367673E 00	2.845
	0.3	4.01	0.1400826E-02	0.1183836E 00	2.845
	6.0	56.22	0.1963951E-01	0.2365115E 01	2.845
	3.0	29.43	0.1028087E-01	0.1182557E 01	2.845
86.0	1.5	14.90	0.5205061E-02	0.5912792E 00	2.845
	0.6	6.05	0.2113465E-02	0.2365110E 00	2.845
	0.3	3.08	0.1075946E-02	0.1182558E 00	2.845
	12.0	82.09	0.2867676E-01	0.4758702E 01	2.845
	6.0	43.72	0.1527284E-01	0.2379351E 01	2.845
	3.0	22.64	0.7908903E-02	0.1189676E 01	2.845
95.0	1.5	11.40	0.3982395E-02	0.5948380E 00	2.845
	0.6	4.62	0.1613919E-02	0.2379352E 00	2.845
	0.3	2.50	0.3733326E-03	0.1189677E 00	2.845
	12.0	64.33	0.2247260E-01	0.4730742E 01	2.845
	6.0	33.71	0.1177602E-01	0.2365371E 01	2.845
	3.0	17.18	0.6001540E-02	0.1182685E 01	2.845
104.0	1.5	8.63	0.3014745E-02	0.5913430E 00	2.845
	0.6	3.55	0.1240132E-02	0.2365371E 00	2.845
	0.3	1.80	0.5287997E-03	0.1182685E 00	2.845
	12.0	51.37	0.1794524E-01	0.4748354E 01	2.845
	6.0	26.50	0.9257328E-02	0.2374178E 01	2.845
	3.0	13.32	0.4653115E-02	0.1187089E 01	2.845
113.0	1.5	6.78	0.2368477E-02	0.5935449E 00	2.845
	0.6	2.77	0.9676528E-03	0.2374179E 00	2.845
	0.3	1.52	0.5309866E-03	0.1187090E 00	2.845
	30.0	93.20	0.3255784E-01	0.1182605E 02	2.845
	12.0	40.42	0.1412004E-01	0.4730419E 01	2.845
	6.0	20.69	0.7227704E-02	0.2365209E 01	2.845
112.0	3.0	10.40	0.3633065E-02	0.1182605E 01	2.845
	1.5	5.20	0.1816533E-02	0.5913027E 00	2.845
	0.6	2.10	0.7335995E-03	0.2365211E 00	2.845
	0.3	1.10	0.3842665E-03	0.1182604E 00	2.845

TABLE A4.15 FLOW CURVE DATA -- SOLATION 12 (CONTD.)

TEMP. °F	SPEED RPM	DIAL RD., %	STRESS LBF/FT2	RATE SEC-1	LEQ. INCH
	30.0	74.33	0.2596594E-01	0.1189985E 02	2.845
	12.0	31.55	0.1102146E-01	0.4759937E 01	2.845
	6.0	15.99	0.5585834E-02	0.2379969E 01	2.845
131.0	3.0	8.03	0.2805144E-02	0.1189984E 01	2.845
	1.5	4.05	0.1414800E-02	0.5949928E 00	2.845
	0.6	1.63	0.5694132E-03	0.2379971E 00	2.845
	0.3	1.00	0.3493330E-03	0.1189985E 00	2.845
	30.0	59.45	0.2076785E-01	0.1192534E 02	2.845
	12.0	25.09	0.3764766E-02	0.4770135E 01	2.845
	6.0	12.62	0.4408583E-02	0.2385066E 01	2.845
140.0	3.0	6.27	0.2190319E-02	0.1192533E 01	2.845
	1.5	3.28	0.1145812E-02	0.5962670E 00	2.845
	0.6	1.40	0.4890664E-03	0.2385067E 00	2.845
	0.3	0.78	0.2724798E-03	0.1192533E 00	2.845
	60.0	84.62	0.2956055E-01	0.2368929E 02	2.845
	30.0	44.90	0.1568505E-01	0.1184466E 02	2.845
	12.0	18.64	0.6511569E-02	0.4737860E 01	2.845
149.0	6.0	9.37	0.3273251E-02	0.2368929E 01	2.845
	3.0	4.78	0.1669812E-02	0.1184464E 01	2.845
	1.5	2.50	0.8733326E-03	0.5922329E 00	2.845
	0.6	0.98	0.3423465E-03	0.2368930E 00	2.845

TABLE A4.16 FLOW CURVE DATA -- SOLUTION 13

TEMP. °F	SPEED RPM	DIAL RD., %	STRESS LBF/FT2	RATE SEC-1	LEQ. INCH
	6.0	94.80	0.1863639E 00	0.2436886E 01	0.511
	3.0	55.18	0.1084764E 00	0.1218442E 01	0.511
77.0	1.5	30.77	0.5048969E-01	0.6092215E 00	0.511
	0.6	13.40	0.2634258E-01	0.2436885E 00	0.511
	0.3	7.00	0.1376105E-01	0.1218443E 00	0.511
	6.0	61.53	0.1209597E 00	0.2392477E 01	0.511
	3.0	34.58	0.5797957E-01	0.1196239E 01	0.511
95.0	1.5	18.60	0.3656510E-01	0.5981197E 00	0.511
	0.6	7.74	0.1521580E-01	0.2392479E 00	0.511
	0.3	4.00	0.7863462E-02	0.1196238E 00	0.511
	12.0	70.18	0.1379644E 00	0.4807691E 01	0.511
	6.0	39.95	0.7853627E-01	0.2403845E 01	0.511
113.0	3.0	21.79	0.4283619E-01	0.1201922E 01	0.511
	1.5	11.37	0.2235188E-01	0.6009615E 00	0.511
	0.6	4.74	0.9318199E-02	0.2403847E 00	0.511
	0.3	2.58	0.5071931E-02	0.1201922E 00	0.511
	30.0	93.13	0.1830810E 00	0.1201910E 02	0.511
	12.0	46.40	0.9121609E-01	0.4807636E 01	0.511
	6.0	25.45	0.5003127E-01	0.2403818E 01	0.511
131.0	3.0	13.40	0.2634258E-01	0.1201909E 01	0.511
	1.5	7.02	0.1380036E-01	0.6009549E 00	0.511
	0.6	2.87	0.5642030E-02	0.2403819E 00	0.511
	0.3	1.53	0.3007773E-02	0.1201909E 00	0.511
	60.0	80.42	0.1584038E 00	0.2446851E 02	0.511
	30.0	50.10	0.9868234E-01	0.1223425E 02	0.511
	12.0	24.87	0.4898663E-01	0.4893699E 01	0.511
149.0	6.0	13.28	0.2615771E-01	0.2446849E 01	0.511
	3.0	6.80	0.1339401E-01	0.1223425E 01	0.511
	1.5	3.60	0.7070949E-02	0.6117127E 00	0.511
	0.6	1.65	0.3250018E-02	0.2446850E 00	0.511

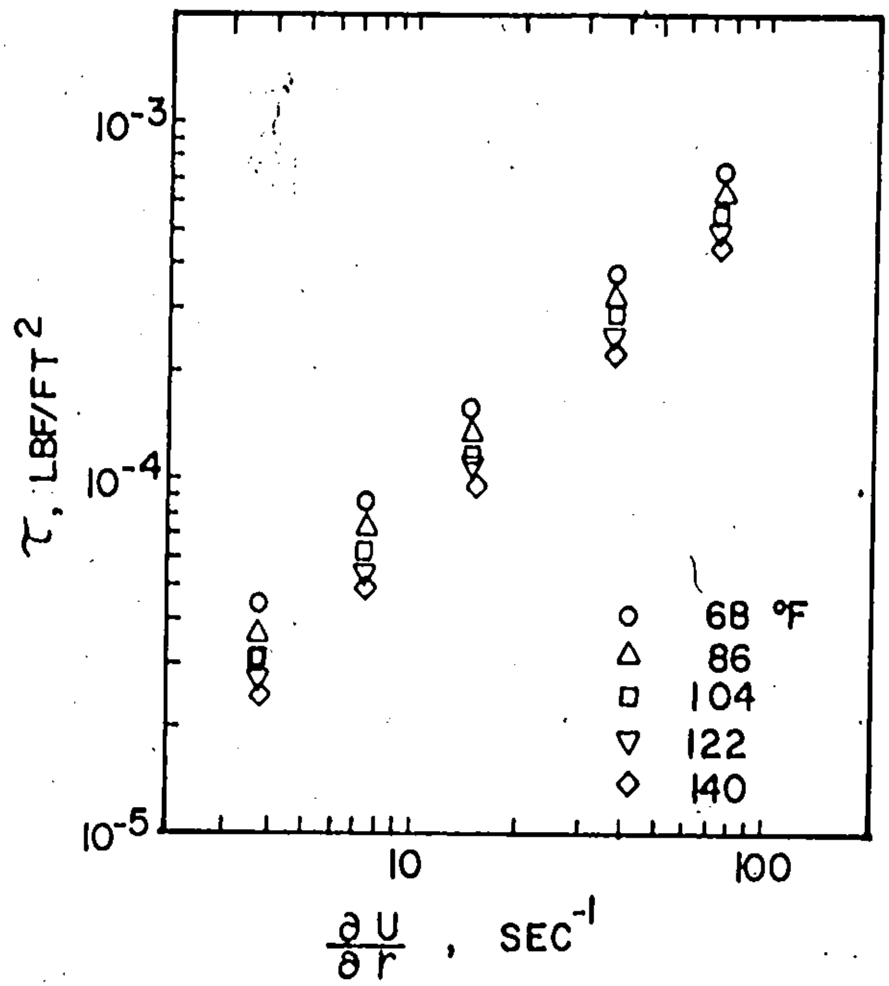


FIG. A4.6 FLOW CURVES OF SOLN. I

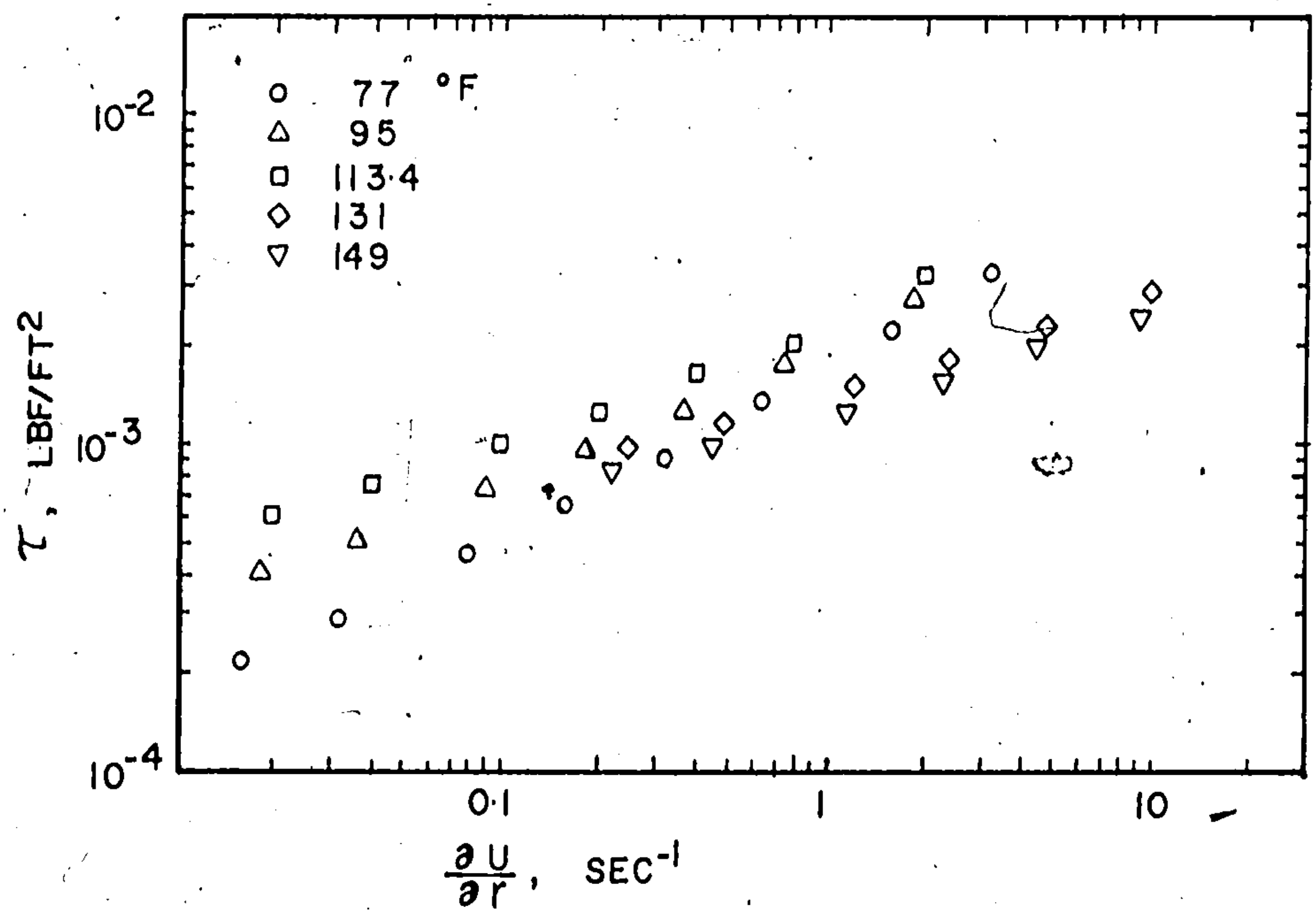


FIG. A4.7 FLOW CURVES OF SOLN. 3

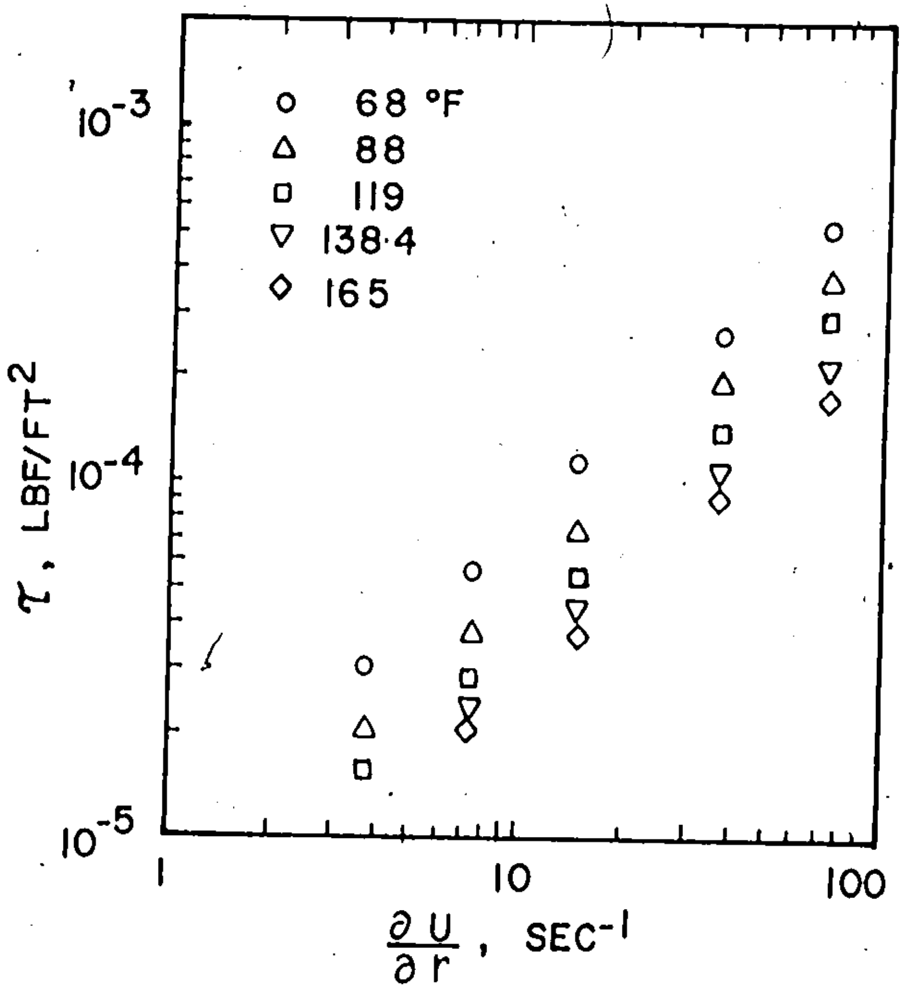


FIG. A4.8 FLOW CURVES OF SOLN. 4

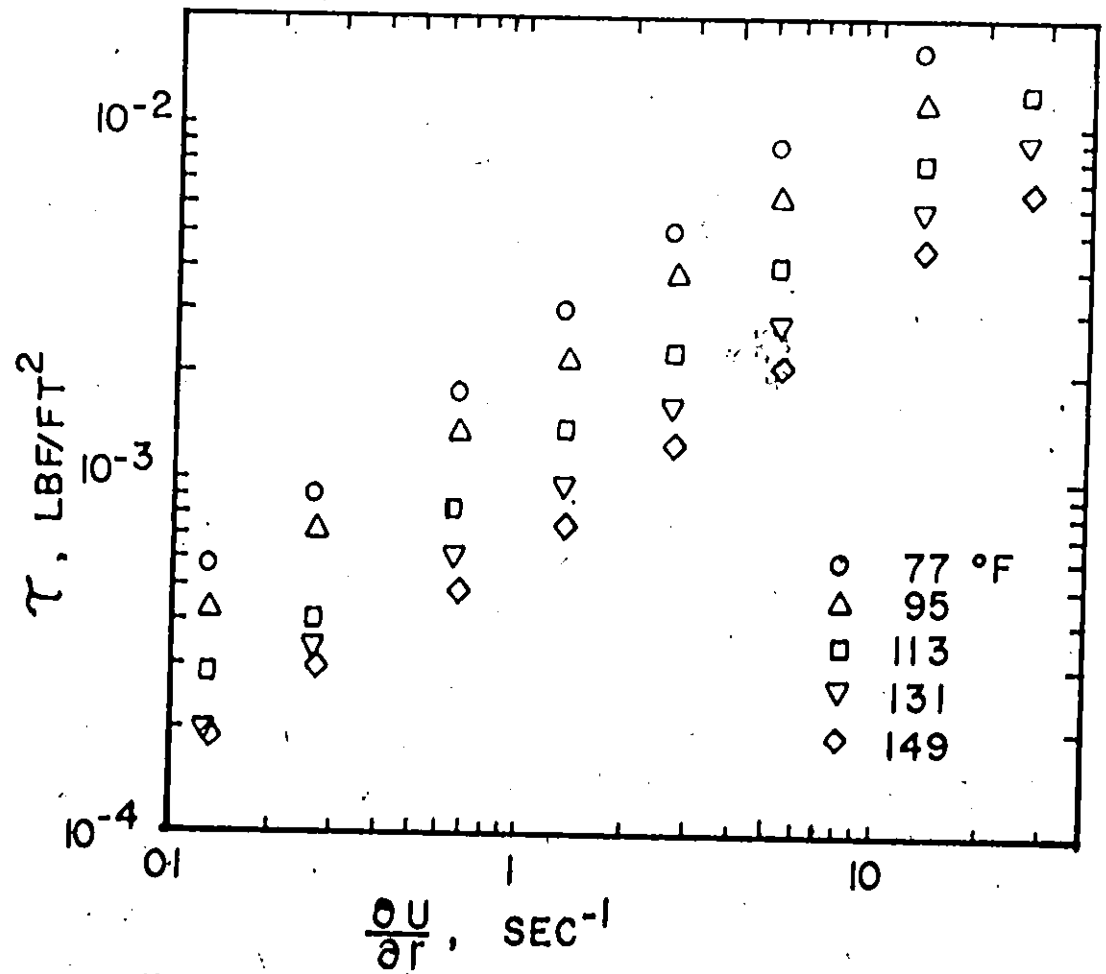


FIG. A4.9 FLOW CURVES OF SOLN. 10

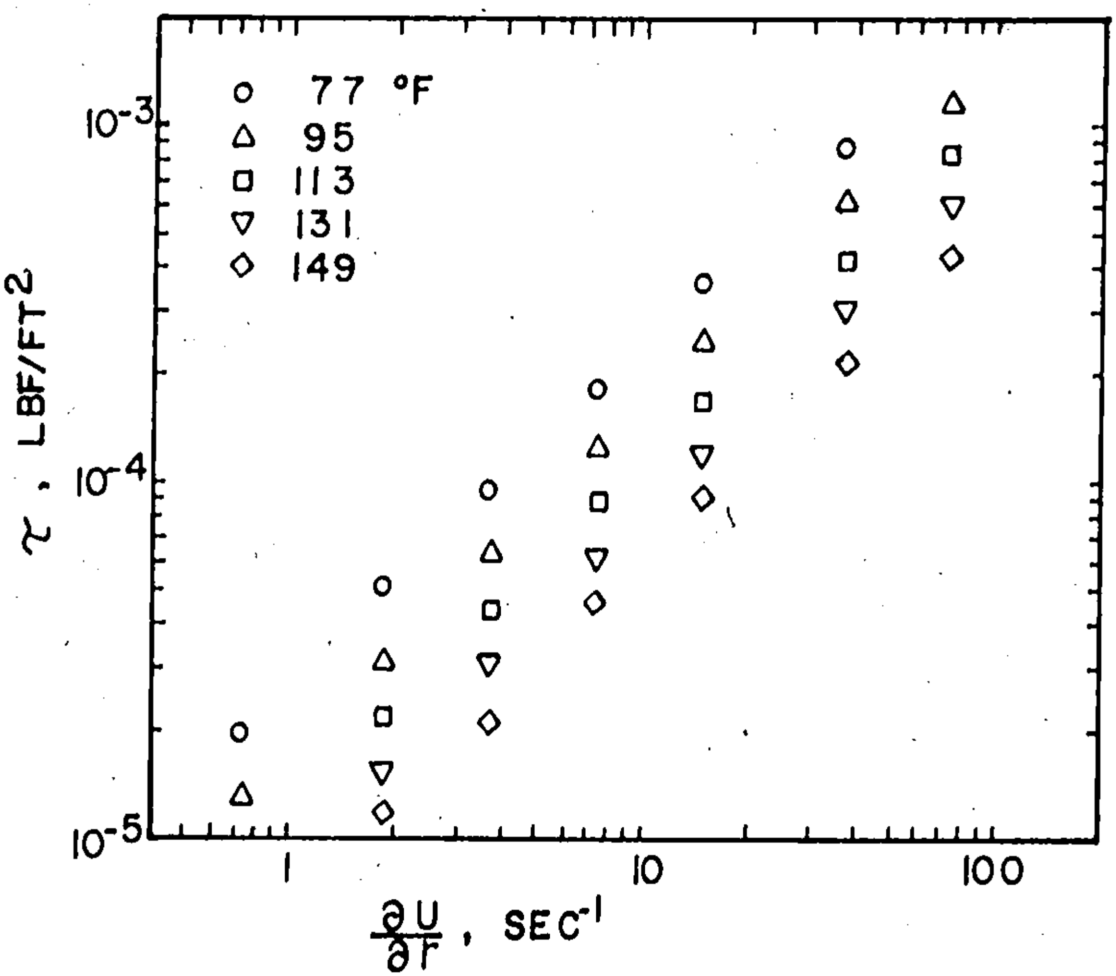


FIG.A4·10 FLOW CURVES OF SOLN. II

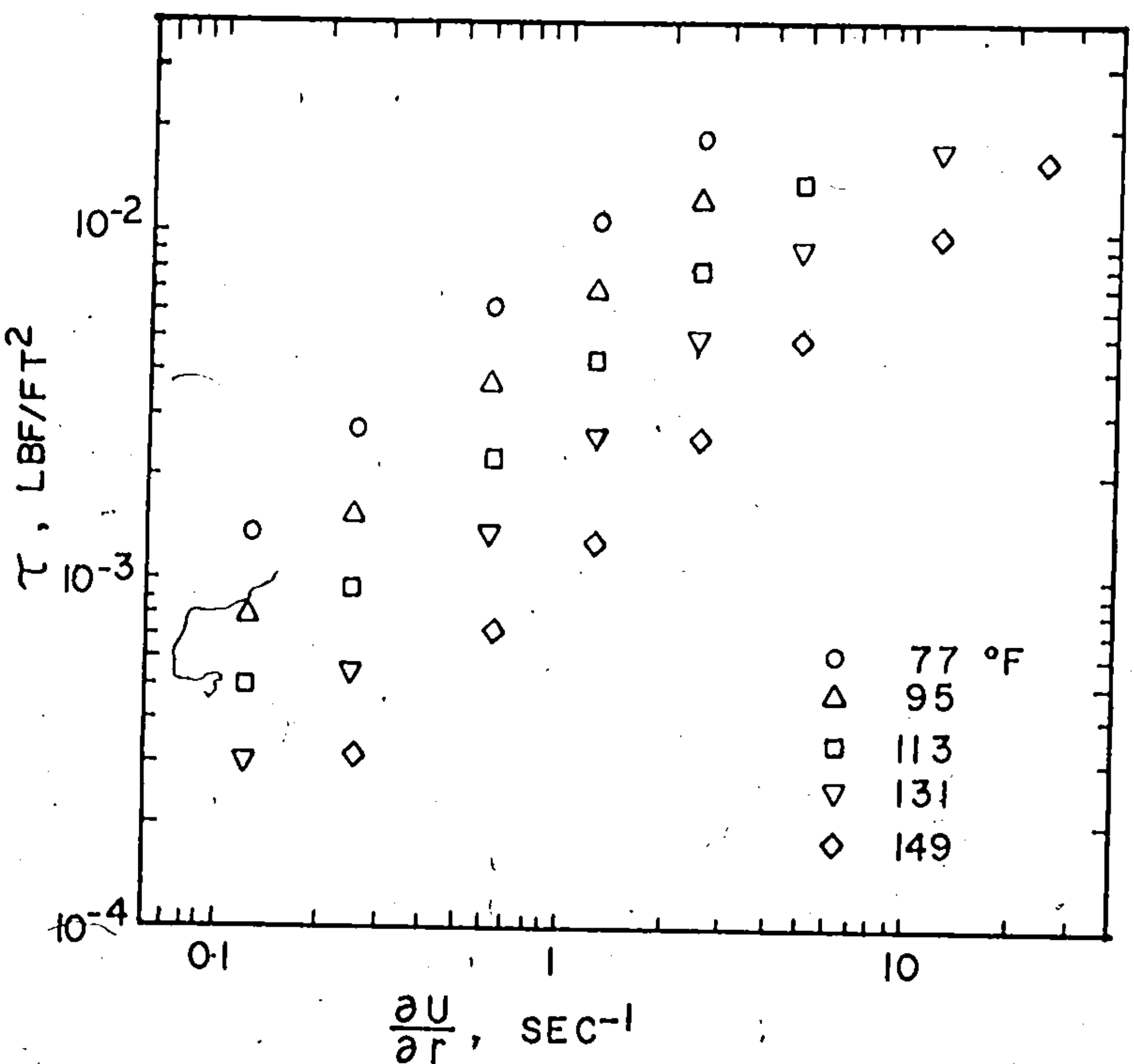


FIG. A4.11 FLOW CURVES OF SOLN. 13

TABLE A4.17

RHEOLOGICAL PROPERTIES OF CARBOPOL 934

T	1.5%		1.0 %		1.5 %	
°F	K*10 LBF.SEC^n/FT^2	n	K*10^4 LBF.SEC^n/FT^2	n	K*10^3 LBF.SEC^n/FT^2	n
68.0	13.273	0.930	7.430	0.824		
77.0	12.101	0.934	8.852	0.795	5.322	0.514
86.0	10.347	0.947	10.060	0.769		
95.0	10.204	0.947	10.995	0.749	7.842	0.417
104.0	9.548	0.948	11.867	0.733		
113.0	9.186	0.946	12.793	0.712	10.452	0.361
122.0	8.040	0.969	13.650	0.693		
131.0	7.781	0.959	14.521	0.682	12.519	0.303
140.0	7.154	0.968	15.329	0.661		
149.0	6.873	0.965	16.254	0.643	14.438	0.282

TABLE A4.18

RHEOLOGICAL PROPERTIES OF CARBOSE 1M

0.25 %			0.5 %			1.0 %		
T °F	K*10^5 LBF.SEC^n/FT^2	n	T °F	K*10^4 LBF.SEC^n/FT^2	n	T °F	K*10^4 LBF.SEC^n/FT^2	n
68.0	8.415	0.968	67.6	2.395	0.916	67.6	10.106	0.906
79.5	7.281	0.990	84.7	1.816	0.916	81.0	7.538	0.920
88.0	5.395	0.995	104.7	1.467	0.894	94.1	5.777	0.932
101.7	4.674	0.991	121.5	1.188	0.890	106.9	4.658	0.920
119.0	4.046	0.994	136.4	1.034	0.877	125.2	3.595	0.904
127.0	4.305	0.934	147.7	0.918	0.869	147.7	2.492	0.905
138.4	3.400	0.971	159.4	0.795	0.872			
154.0	3.529	0.923	166.3	0.877				
165.0	3.282	0.930						

TABLE A4.19

RHEOLOGICAL PROPERTIES OF CARBOSE IM (WITH 0.1 SODIUM BANZOATE)

T	0.5%		1.0%		1.5%		2.5%	
°F	$K*10^4$ $\frac{LBF.SEC^n}{FT^2}$	n	$K*10^4$ $\frac{LBF.SEC^n}{FT^2}$	n	$K*10^3$ $\frac{LBF.SEC^n}{FT^2}$	n	$K*10^2$ $\frac{LBF.SEC^n}{FT^2}$	n
68					3.572	0.863		
68	2.263	0.950	8.581	0.913	3.715	0.842		
77	1.973	0.944	7.257	0.909	2.905	0.869	2.507	0.745
86	1.510	0.952	5.774	0.930	2.540	0.828		
95	1.374	0.963	4.950	0.927	1.992	0.875	1.867	0.724
104	1.240	0.956	4.260	0.926	1.796	0.849		
113	1.090	0.953	3.582	0.934	1.413	0.886	1.177	0.737
122	0.874	0.980	3.110	0.935				
131	0.795	0.980	2.627	0.945	1.147	0.850	0.857	0.730
140	0.773	0.960	2.356	0.937				
149	0.735	0.950	2.117	0.934	0.990	0.814	0.702	0.682

TABLE A4.20

RHEOLOGICAL PROPERTIES OF NATROSOL 250H

T	0.25%		0.75%		1.25%	
°F	$K*10^4$ $\frac{LBF.SEC^n}{FT^2}$	n	$K*10^3$ $\frac{LBF.SEC^n}{FT^2}$	n	$K*10^2$ $\frac{LBF.SEC^n}{FT^2}$	n
68.0	3.237	0.979	14.697	0.957		
77.0	2.698	0.972	11.264	0.969	8.934	0.873
86.0	2.284	0.962	8.617	0.973		
95.0	1.762	0.980	6.568	0.956	5.642	0.917
104.0	1.502	0.977	5.038	0.972		
113.0	1.130	0.985	3.979	0.962	3.480	0.905
122.0	1.027	0.992	3.049	0.972		
131.0	.871	0.992	2.422	0.954	2.127	0.905
140.0	.722	0.999	2.079	0.949		
149.0	.646	0.990	1.419	0.968	1.656	0.859

APPENDIX V

HEAT TRANSFER DATA

Note: All data in this Appendix were evaluated with the characteristic length equal to the radius.

Table A5.1 Heat Transfer Data -- Water

Run	T_B °F	ΔT °F	Q Btu/hr	$\overline{h}$ Btu/hrft²°F	$\overline{N_{Nu}}$	N_{Gr}	Z_1	N_{Pr}	Z_2	Z
1	150.4	31.98	1613.3	94.6	52.35	93722090.0	98.39	3.14	1.33	131.03
2	131.5	22.81	1053.8	86.7	48.92	46488220.0	82.57	3.63	1.38	113.94
3	146.3	23.00	1601.9	130.6	72.28	67977370.0	90.80	3.13	1.33	120.79
4	149.8	25.88	1601.5	116.1	64.09	80414220.0	94.70	3.07	1.32	125.37
5	146.9	26.22	1602.0	114.6	63.44	75620750.0	93.25	3.16	1.33	124.33
6	79.8	2.62	58.4	41.8	24.71	1230468.0	33.31	6.04	1.57	52.21
7	84.6	7.11	223.0	58.8	34.67	3750940.0	44.01	5.82	1.55	68.34
8	92.5	12.15	507.2	78.3	45.75	8123504.0	53.39	5.39	1.52	81.33
9	93.6	12.65	507.2	75.2	43.88	8747750.0	54.38	5.33	1.52	82.62
10	71.4	0.42	5.9	26.3	15.72	137281.5	19.25	6.71	1.61	30.99
11	72.9	1.63	33.6	38.7	23.14	562609.7	27.39	6.62	1.60	43.93
12	75.4	3.83	93.7	45.9	27.38	1422827.0	34.54	6.49	1.60	55.12
13	79.1	6.00	181.7	56.8	33.74	2498273.0	39.76	6.26	1.58	62.89
14	86.3	9.36	319.8	64.1	37.73	5033347.0	47.37	5.78	1.55	73.45
15	92.7	12.79	491.7	72.1	42.16	8449695.0	53.92	5.41	1.52	82.21
16	99.0	15.23	632.6	77.9	45.21	12431390.0	59.38	5.03	1.50	88.93
17	104.9	18.12	807.4	83.6	48.23	17135610.0	64.34	4.78	1.48	95.15
18	107.2	18.16	807.4	83.4	47.99	18747020.0	65.80	4.63	1.47	96.53
19	115.7	22.05	1069.0	90.9	51.90	28075530.0	72.79	4.29	1.44	104.78
20	115.8	21.90	1069.0	91.6	52.25	28364110.0	72.98	4.27	1.44	104.88
21	122.5	25.01	1337.1	100.3	56.84	37364990.0	78.18	4.05	1.42	110.89
22	131.1	29.14	1703.5	109.7	61.74	53331720.0	85.46	3.75	1.39	118.92
23	76.5	3.69	86.4	43.9	26.14	1450535.0	34.70	6.39	1.59	55.18
24	78.3	4.77	134.1	52.8	31.34	1979059.0	37.51	6.26	1.58	59.34
25	81.9	7.08	219.9	58.3	34.45	3313697.0	42.67	6.04	1.57	66.88
26	89.6	12.14	403.4	62.3	36.57	7117418.0	51.65	5.62	1.54	79.54
27	96.2	13.76	540.6	73.7	42.88	10428090.0	56.83	5.16	1.51	85.64
28	102.2	15.53	698.3	84.4	48.74	14062680.0	61.24	4.84	1.48	90.85
29	111.9	20.45	947.4	86.9	49.78	23689800.0	69.77	4.45	1.45	101.31
30	120.4	23.77	1216.7	96.0	54.56	26628040.0	71.83	4.66	1.47	105.52
31	127.9	27.11	1446.8	100.1	57.13	46636140.0	82.64	3.88	1.40	116.01

$Z_1 = (N_{Gr})^{1/4}$; $Z_2 = (N_{Pr})^{1/4}$; $Z = Z_1 Z_2$

Table A5.1 Heat Transfer Data -- Water (Contd.)

Run	T_B °F	ΔT °F	Q Btu/hr	$\bar{h}$ Btu/hrft²°F	$\overline{N_{Nu}}$	N_{Gr}	Z_1	N_{Pr}	Z_2	Z
32	74.2	0.36	5.2	27.0	16.08	136469.5	19.22	6.45	1.59	30.64
33	73.3	0.37	4.9	24.7	14.73	135106.3	19.17	6.52	1.60	30.64
34	73.6	0.60	8.4	26.2	15.63	221195.8	21.69	6.49	1.60	34.62
35	73.8	1.00	15.1	28.4	16.92	368659.7	24.64	6.49	1.60	39.33
36	74.7	1.43	24.3	31.9	19.03	542086.9	27.13	6.45	1.59	43.25
37	75.2	1.65	31.1	35.4	21.09	633234.5	28.21	6.42	1.59	44.91
38	76.3	2.87	61.1	39.9	23.76	1133936.0	32.63	6.36	1.59	51.82
39	77.6	4.07	101.8	46.9	27.84	1667458.0	35.93	6.28	1.58	56.89
40	79.4	4.73	122.2	48.5	28.75	2049048.0	37.83	6.20	1.58	59.69
41	79.7	5.42	144.1	49.9	29.58	2347958.0	39.14	6.20	1.58	61.76
42	81.0	5.94	167.1	52.8	31.25	2726257.0	40.63	6.07	1.57	63.79
43	82.5	7.19	205.8	53.7	31.74	3432023.0	43.04	6.01	1.57	67.38
44	84.4	8.13	254.8	58.8	34.69	4099599.0	45.00	5.91	1.56	70.15
45	87.0	9.62	321.1	62.6	36.83	5275602.0	47.93	5.72	1.55	74.12
46	90.7	11.11	396.5	66.9	39.20	6950622.0	51.35	5.50	1.53	78.63
47	94.6	12.68	480.5	71.1	41.42	9136087.0	54.98	5.25	1.51	83.21
48	99.0	14.47	583.3	75.6	43.88	11811030.0	58.62	5.03	1.50	87.80
49	102.4	16.39	685.2	78.4	45.31	14841430.0	62.07	4.84	1.48	92.08
50	104.3	17.51	746.8	80.0	46.16	16473840.0	63.71	4.78	1.48	94.21
51	106.8	19.47	806.0	77.6	44.74	19348760.0	66.32	4.69	1.47	97.62
52	109.7	19.88	892.0	84.2	48.33	21424640.0	68.03	4.57	1.46	99.45
53	114.3	21.76	1015.0	87.5	49.99	26764640.0	71.93	4.35	1.44	103.89
54	119.7	25.61	1140.9	84.1	47.89	35239480.0	77.05	4.17	1.43	110.12
55	122.9	26.55	1336.1	94.4	53.54	48334880.0	83.38	3.67	1.38	115.40
56	126.4	28.24	1525.0	101.3	57.26	45847040.0	82.29	3.93	1.41	115.84
57	131.6	30.65	1698.8	104.0	58.53	56095290.0	86.54	3.75	1.39	120.44
58	134.3	30.34	1702.2	105.2	59.09	60205450.0	88.09	3.63	1.38	121.61
59	148.2	30.88	1923.9	116.9	64.74	87113720.0	96.61	3.19	1.34	129.10
60	155.3	30.70	1948.2	119.0	65.68	102407700.0	100.60	2.99	1.32	132.30
61	160.5	31.87	2046.4	120.4	66.11	118117800.0	104.25	2.87	1.30	135.70
62	161.5	31.54	2116.2	125.9	68.99	120159300.0	104.70	2.84	1.30	135.93
63	160.4	30.50	1975.9	121.5	66.65	115864900.0	103.75	2.84	1.30	134.72

Table A5.2 Heat Transfer Data -- Soln. 1

Run	T_s °F	ΔT °F	Q Btu/hr	$\overline{h}$ Btu/hrft^2°F	$\overline{N_{Nu}}$	N_{Gr}	Z_1	N_{Pr}	Z_2	Z
1	73.7	0.49	4.7	18.1	10.85	0.2544422E 04	8.15	0.425816E 02	2.47	20.13
2	74.5	1.12	15.6	26.2	15.56	0.6605121E 04	10.53	0.387953E 02	2.41	25.41
3	76.0	2.11	34.7	30.8	18.41	0.1410238E 05	12.90	0.357977E 02	2.37	30.54
4	77.4	3.43	60.7	33.2	19.81	0.2527158E 05	15.09	0.336283E 02	2.33	35.19
5	79.6	4.84	101.4	39.3	23.45	0.3952333E 05	17.03	0.318003E 02	2.30	39.17
6	81.3	6.20	145.0	43.9	26.15	0.5457687E 05	18.58	0.305246E 02	2.28	42.31
7	83.6	8.13	194.2	44.8	26.67	0.7824081E 05	20.48	0.291047E 02	2.25	46.10
8	87.8	9.91	256.1	48.5	28.78	0.1109918E 06	22.55	0.272325E 02	2.21	49.93
9	90.3	11.94	329.7	51.8	30.72	0.1444940E 06	24.23	0.261721E 02	2.19	53.14
10	93.0	14.13	402.1	53.4	31.62	0.1855433E 06	25.95	0.251286E 02	2.17	56.33
11	96.2	16.02	489.8	57.4	33.91	0.2314270E 06	27.58	0.240443E 02	2.15	59.24
12	99.6	18.18	587.5	60.6	35.78	0.2890132E 06	29.33	0.229850E 02	2.12	62.30
13	102.9	20.25	679.6	62.9	37.09	0.3521072E 06	30.99	0.220330E 02	2.10	65.14
14	106.6	22.57	783.6	65.1	38.30	0.4337464E 06	32.85	0.210132E 02	2.08	68.25
15	110.9	24.59	891.3	68.0	39.89	0.5282866E 06	34.74	0.199494E 02	2.05	71.25
16	116.3	27.29	1029.0	70.7	41.38	0.6718904E 06	37.20	0.186926E 02	2.02	75.08
17	127.4	28.35	1150.9	76.1	44.22	0.9350886E 06	41.06	0.16[illegible]113E 02	1.95	80.12
18	131.2	31.53	1283.7	76.4	44.28	0.1117933E 07	43.18	0.156733E 02	1.93	83.44
19	135.8	33.69	1432.9	78.8	46.16	0.1314000E 07	45.25	0.149127E 02	1.91	86.39
20	139.4	35.31	1574.8	85.7	48.32	0.1479599E 07	46.85	0.143580E 02	1.89	88.61
21	146.5	38.19	1715.7	84.3	48.51	0.1832543E 07	49.92	0.133512E 02	1.86	92.73
22	151.9	39.52	1812.5	86.0	49.39	0.2099366E 07	52.04	0.126377E 02	1.83	95.36
23	157.3	38.79	1789.1	86.5	49.53	0.2279442E 07	53.51	0.119763E 02	1.81	96.76
24	160.5	40.86	1917.7	88.0	50.35	0.2513268E 07	55.07	0.116516E 02	1.80	98.91

$Z_1 = N_{Gr}^{\frac{1}{2(n+1)}}$; $Z_2 = N_{Pr}^{\frac{n}{3n+1}}$; $Z = Z_1 Z_2$

Table A5.3 Heat Transfer Data -- Soln. 2

Run	T_s °F	ΔT °F	Q Btu/hr	$\bar{h}$ Btu/hrft^2°F	$\overline{N_{Nu}}$	N_{Gr}	Z_1	N_{Pr}	Z_2	Z
1	82.0	4.02	46.9	21.9	13.02	0.5045642E 03	5.70	0.269324E 03	3.71	21.15
2	84.2	6.11	81.4	25.0	14.85	0.8244863E 03	6.56	0.252884E 03	3.65	23.94
3	86.4	8.36	121.5	27.3	16.20	0.1189337E 04	7.29	0.241368E 03	3.61	26.28
4	89.5	10.73	168.5	29.4	17.47	0.1575760E 04	7.93	0.234102E 03	3.57	28.32
5	92.5	13.39	227.6	31.9	18.89	0.2029939E 04	8.55	0.227130E 03	3.54	30.27
6	96.7	16.29	293.9	33.8	20.00	0.2518035E 04	9.15	0.222148E 03	3.51	32.14
7	100.0	19.23	361.8	35.3	20.83	0.3044779E 04	9.70	0.216804E 03	3.49	33.83
8	104.0	21.49	444.3	38.8	22.84	0.3421687E 04	10.11	0.214330E 03	3.47	35.05
9	108.5	25.86	536.8	38.9	22.90	0.4248383E 04	10.83	0.207538E 03	3.43	37.15
10	116.2	28.64	629.5	41.2	24.13	0.4684215E 04	11.34	0.204972E 03	3.40	38.55
11	120.9	31.94	727.1	42.7	24.94	0.5313199E 04	11.87	0.200240E 03	3.37	40.00
12	126.3	36.47	845.9	43.5	25.36	0.6227547E 04	12.55	0.193964E 03	3.33	41.83
13	131.1	39.97	959.5	45.0	26.19	0.6943973E 04	13.08	0.189197E 03	3.30	43.20
14	136.5	43.80	1081.3	46.3	26.87	0.7751133E 04	13.66	0.184026E 03	3.27	44.65
15	144.7	47.33	1218.7	48.3	27.91	0.8468020E 04	14.32	0.177984E 03	3.22	46.12
16	151.1	51.04	1348.2	49.6	28.55	0.9273998E 04	14.94	0.172380E 03	3.18	47.51
17	155.8	49.67	1285.0	48.5	27.38	0.8859777E 04	15.02	0.171051E 03	3.15	47.37
18	161.2	53.74	1429.3	49.9	28.51	0.9768805E 04	15.64	0.165522E 03	3.12	48.79
19	166.2	57.27	1578.7	51.7	29.61	0.1056049E 05	16.20	0.160772E 03	3.09	50.00

Table A5.4 Heat Transfer Data -- Soln. 3

Run	T_s °F	ΔT °F	Q Btu/hr	$\bar{h}$ Btu/hrft²°F	$\overline{N_{Nu}}$	N_{Gr}	Z_1	N_{Pr}	Z_2	Z
1	76.5	1.11	3.7	6.3	3.78	0.1156423E 01	1.05	0.403773E 04	5.39	5.65
2	78.1	3.06	15.2	9.3	5.54	0.5063620E 01	1.71	0.254130E 04	4.89	8.35
3	82.3	7.14	34.7	9.1	5.43	0.1634015E 02	2.53	0.179801E 04	4.49	11.38
4	86.4	10.71	60.9	10.7	6.34	0.2772763E 02	3.05	0.154856E 04	4.30	13.09
5	92.4	16.27	101.6	11.7	6.95	0.4760570E 02	3.71	0.132031E 04	4.08	15.13
6	97.6	21.36	147.0	12.9	7.64	0.6750116E 02	4.23	0.118381E 04	3.93	16.62
7	104.3	26.88	195.3	13.6	8.04	0.8868904E 02	4.74	0.108436E 04	3.77	17.87
8	110.9	32.56	257.1	14.8	8.71	0.1119423E 03	5.24	0.997084E 03	3.63	19.02
9	118.5	39.52	329.5	15.6	9.17	0.1429366E 03	5.83	0.904303E 03	3.48	20.31
10	128.6	46.03	403.2	16.4	9.59	0.1679998E 03	6.40	0.835799E 03	3.30	21.14
11	136.9	53.37	490.8	17.3	10.04	0.2038352E 03	7.03	0.759974E 03	3.17	22.27
12	146.0	61.00	584.7	18.0	10.43	0.2425326E 03	7.68	0.691955E 03	3.04	23.34
13	155.4	68.90	679.9	18.5	10.70	0.2850291E 03	8.36	0.631124E 03	2.92	24.39
14	165.9	77.53	784.1	19.0	10.92	0.3335376E 03	9.10	0.574664E 03	2.80	25.48
15	172.8	81.89	886.2	20.3	11.66	0.3551709E 03	9.48	0.549689E 03	2.73	25.92
16	183.5	89.55	1015.4	21.3	12.17	0.3969822E 03	10.10	0.512747E 03	2.65	26.76

Table A5.5 Heat Transfer Data -- Soln. 4

Run	T_B °F	ΔT °F	Q Btu/hr	$\bar{h}$ Btu/hr ft^2°F	$\overline{N_{Nu}}$	N_{Gr}	Z_1	N_{Pr}	Z_2	Z
1	79.3	1.74	31.4	33.8	20.32	0.6317129E 05	16.12	0.214600E 02	2.15	34.61
2	77.2	2.08	34.9	31.4	18.93	0.6601006E 05	16.26	0.225164E 02	2.17	35.35
3	79.9	4.76	98.4	38.8	23.32	0.1654518E 06	20.51	0.217074E 02	2.15	44.18
4	84.1	7.77	189.9	45.9	27.51	0.3176579E 06	24.24	0.203876E 02	2.12	51.37
5	88.9	10.40	284.0	51.2	30.57	0.5160993E 06	27.48	0.189011E 02	2.08	57.14
6	94.4	13.14	394.7	56.3	33.64	0.8042532E 06	30.87	0.173812E 02	2.04	62.83
7	99.2	15.01	493.4	61.7	36.72	0.1100697E 07	33.55	0.161483E 02	2.00	67.02
8	103.6	17.00	583.1	64.3	38.23	0.1447649E 07	36.10	0.151755E 02	1.97	70.97
9	99.0	20.95	747.6	67.3	40.13	0.1336817E 07	35.12	0.170110E 02	2.02	71.09
10	110.3	23.27	878.0	70.8	41.96	0.2283206E 07	40.67	0.142706E 02	1.94	78.72
11	123.6	26.75	1075.3	75.4	44.42	0.3762732E 07	46.81	0.122149E 02	1.86	87.06
12	127.9	27.05	1139.5	79.0	46.42	0.4178520E 07	48.31	0.117189E 02	1.84	88.[illegible]
13	134.0	29.14	1313.6	84.6	49.54	0.4977054E 07	50.83	0.111907E 02	1.82	92.43
14	141.1	31.12	1418.6	85.5	49.96	0.5847640E 07	53.37	0.106945E 02	1.80	95.90
15	145.8	33.22	1581.6	89.3	52.10	0.6564910E 07	55.23	0.104272E 02	1.79	98.59
16	151.8	35.47	1724.5	91.2	53.10	0.7433239E 07	57.37	0.101128E 02	1.77	101.59
17	159.0	37.27	1896.7	95.5	55.48	0.8377788E 07	59.64	0.973252E 01	1.75	104.56
18	157.2	35.43	1769.6	93.7	54.46	0.7872197E 07	58.63	0.980060E 01	1.76	102.97
19	147.7	29.84	1378.4	86.6	50.48	0.6115418E 07	54.48	0.102507E 02	1.78	96.82
20	115.7	14.21	492.9	65.1	38.35	0.1879950E 07	39.17	0.126256E 02	1.88	73.47
21	97.3	10.02	283.2	53.0	31.56	0.7499202E 06	30.48	0.160615E 02	1.99	60.79
22	87.7	4.53	102.3	42.4	25.33	0.2427214E 06	22.77	0.184351E 02	2.07	47.03
23	79.4	0.91	14.3	29.5	17.73	0.3369720E 05	13.77	0.213703E 02	2.15	29.53
24	79.6	2.05	34.1	31.2	18.75	0.7523594E 05	16.84	0.213589E 02	2.14	36.13

Table A5.6 Heat Transfer Data -- Soln. 5

Run	T_s °F	ΔT °F	Q Btu/hr	$\bar{h}$ Btu/hrft^2°F	$\overline{N_{Nu}}$	N_{Gr}	Z_1	N_{Pr}	Z_2	Z
1	76.3	0.88	8.3	17.7	10.55	0.2206246E 04	7.49	0.699999E 02	2.82	21.12
2	76.3	1.26	12.9	19.3	11.60	0.3221057E 04	8.26	0.685984E 02	2.81	23.20
3	75.1	1.87	23.3	23.4	14.10	0.4609273E 04	9.07	0.685472E 02	2.81	25.44
4	76.1	2.71	37.5	26.0	15.64	0.7105559E 04	10.16	0.661066E 02	2.78	28.26
5	77.4	3.44	52.8	28.8	17.32	0.9593207E 04	10.99	0.640492E 02	2.76	30.35
6	79.2	5.44	98.7	34.1	20.49	0.1646568E 05	12.67	0.610888E 02	2.73	34.56
7	81.3	8.89	140.1	29.6	17.78	0.2865302E 05	14.65	0.586102E 02	2.70	39.56
8	88.0	8.84	193.5	41.1	24.58	0.3873856E 05	15.92	0.519689E 02	2.62	41.73
9	90.4	10.80	254.3	44.2	26.42	0.5148447E 05	17.17	0.498842E 02	2.59	44.55
10	97.1	14.40	377.5	49.2	29.31	0.8670888E 05	19.76	0.448653E 02	2.53	49.92
11	99.4	15.51	425.0	51.4	30.61	0.1005346E 06	20.57	0.433901E 02	2.51	51.53
12	102.1	17.12	489.0	53.6	31.87	0.1209009E 06	21.63	0.417029E 02	2.48	53.64
13	107.6	18.69	555.7	55.8	33.07	0.1595667E 06	23.35	0.383238E 02	2.43	56.71
14	110.0	19.51	601.8	57.9	34.27	0.1797508E 06	24.14	0.370171E 02	2.41	58.10
15	113.3	20.46	644.6	59.1	34.94	0.2087074E 06	25.16	0.353359E 02	2.38	59.87
16	115.0	21.40	693.3	60.8	35.90	0.2287564E 06	25.80	0.345549E 02	2.37	61.05
17	116.4	22.30	729.1	61.3	36.21	0.2472115E 06	26.36	0.339497E 02	2.36	62.09
18	118.2	23.36	776.2	62.3	36.77	0.2722008E 06	27.07	0.331454E 02	2.34	63.38
19	121.0	24.93	846.6	63.7	37.54	0.3123878E 06	28.12	0.320031E 02	2.32	65.26
20	125.2	26.48	936.6	66.4	39.03	0.3715778E 06	29.52	0.303280E 02	2.29	67.60
21	128.9	28.12	1032.7	68.9	40.46	0.4336313E 06	30.83	0.289737E 02	2.26	69.80
22	132.8	29.90	1124.2	70.5	41.36	0.5070571E 06	32.22	0.276555E 02	2.24	72.09
23	137.4	32.68	1277.4	73.3	42.93	0.6132456E 06	33.98	0.262805E 02	2.21	75.07
24	142.3	34.35	1373.6	75.0	43.83	0.7263646E 06	35.67	0.247735E 02	2.18	77.65
25	146.6	36.72	1517.7	77.5	45.24	0.8523254E 06	37.33	0.236203E 02	2.15	80.29
26	151.7	38.28	1627.9	79.8	46.47	0.1000440E 07	39.11	0.222452E 02	2.12	82.87
27	155.8	40.19	1753.0	81.8	47.62	0.1149306E 07	40.70	0.212300E 02	2.09	85.24
28	160.5	42.14	1890.2	84.1	48.91	0.1340471E 07	42.55	0.200826E 02	2.07	87.90

Table A5.7 Heat Transfer Data -- Soln. 6

Run	T_s °F	ΔT °F	Q Btu/hr	$\bar{h}$ Btu/hrft^2°F	$\overline{N_{Nu}}$	N_{Gr}	Z_1	N_{Pr}	Z_2	Z
1	73.8	0.62	4.9	14.7	8.95	0.8413962E 02	3.17	0.324114E 03	4.11	13.05
2	73.9	0.72	4.9	12.6	7.52	0.9925151E 02	3.31	0.320637E 03	4.10	13.59
3	75.6	1.90	15.5	15.3	9.24	0.3024688E 03	4.43	0.294334E 03	4.02	17.80
4	76.9	2.91	26.3	17.0	10.21	0.5024504E 03	5.06	0.281548E 03	3.97	20.10
5	79.1	4.45	41.2	17.4	10.44	0.8614985E 03	5.82	0.265936E 03	3.92	22.81
6	80.6	5.48	57.6	19.7	11.85	0.1145361E 04	6.27	0.256548E 03	3.88	24.35
7	82.7	7.07	86.4	22.9	13.77	0.1619167E 04	6.87	0.245647E 03	3.84	26.38
8	83.9	7.97	85.2	20.1	12.04	0.1925129E 04	7.18	0.239699E 03	3.82	27.43
9	84.9	8.42	108.8	24.2	14.54	0.2131541E 04	7.38	0.234937E 03	3.80	28.03
10	85.9	9.38	130.5	26.1	15.65	0.2473527E 04	7.67	0.230406E 03	3.78	29.00
11	88.3	10.22	153.9	28.3	16.92	0.3025566E 04	8.09	0.219491E 03	3.74	30.21
12	90.8	11.80	185.7	29.5	17.66	0.3885597E 04	8.63	0.209360E 03	3.69	31.89
13	92.8	13.17	219.6	31.3	18.69	0.4714711E 04	9.08	0.201685E 03	3.66	33.24
14	95.8	15.22	271.4	33.4	19.97	0.6116016E 04	9.72	0.191486E 03	3.61	35.13
15	98.9	17.43	331.2	35.6	21.24	0.7923262E 04	10.41	0.181166E 03	3.56	37.09
16	102.2	18.95	378.6	37.5	22.30	0.9945051E 04	11.05	0.170100E 03	3.51	38.77
17	105.5	20.94	439.3	39.4	23.38	0.1250520E 05	11.73	0.160495E 03	3.46	40.59
18	109.5	22.36	491.3	41.2	24.44	0.1595743E 05	12.51	0.148307E 03	3.39	42.44
19	113.4	24.80	585.3	44.3	26.21	0.2056587E 05	13.38	0.138445E 03	3.34	44.61
20	117.0	26.41	652.0	46.3	27.37	0.2547904E 05	14.15	0.129232E 03	3.28	46.41
21	121.4	28.53	729.9	48.0	28.30	0.3276197E 05	15.12	0.119270E 03	3.21	48.62
22	124.7	30.36	805.4	49.8	29.31	0.3942229E 05	15.88	0.112629E 03	3.17	50.34
23	130.4	31.87	884.0	52.0	30.56	0.5282189E 05	17.16	0.100627E 03	3.08	52.91
24	134.9	34.73	1002.9	54.2	31.77	0.6677538E 05	18.26	0.937568E 02	3.03	55.32
25	141.3	36.72	1119.7	57.2	33.46	0.8963563E 05	19.74	0.838183E 02	2.95	58.19
26	147.8	39.87	1267.2	59.6	34.80	0.1190536E 06	21.29	0.761497E 02	2.88	61.28
27	151.1	41.69	1364.6	61.4	35.80	0.1359918E 06	22.05	0.729940E 02	2.85	62.82
28	155.5	43.24	1478.3	64.1	37.34	0.1590231E 06	22.99	0.689802E 02	2.81	64.60
29	161.4	45.70	1574.1	64.6	37.57	0.1912333E 06	24.16	0.649178E 02	2.77	66.86
30	164.7	47.99	1663.2	65.0	37.78	0.2113823E 06	24.81	0.633462E 02	2.75	68.25
31	165.0	47.54	1670.4	65.9	38.30	0.2112858E 06	24.81	0.631134E 02	2.75	68.19

Table A5.8 Heat Transfer Data -- Soln. 7

Run	T_s °F	ΔT °F	Q Btu/hr	$\overline{h}$ Btu/hrft^2°F	$\overline{N_{Nu}}$	N_{Gr}	Z_1	N_{Pr}	Z_2	Z
1	73.5	0.45	4.4	18.5	11.16	0.1318661E 04	6.30	0.703638E 02	2.86	18.00
2	75.8	2.52	33.5	25.0	15.05	0.8619379E 04	10.18	0.643484E 02	2.80	28.47
3	77.9	3.91	60.7	29.1	17.54	0.1477184E 05	11.69	0.615475E 02	2.77	32.32
4	79.8	5.56	100.2	33.8	20.33	0.2267509E 05	13.04	0.594559E 02	2.74	35.75
5	82.1	7.19	145.6	38.0	22.83	0.3220632E 05	14.26	0.571699E 02	2.72	38.72
6	85.6	8.75	197.0	42.2	25.33	0.4569592E 05	15.58	0.538199E 02	2.68	41.70
7	88.3	11.06	256.6	43.5	26.07	0.6351314E 05	16.94	0.517263E 02	2.65	44.90
8	91.3	13.17	340.4	48.5	29.00	0.8461600E 05	18.22	0.494367E 02	2.62	47.76
9	94.3	15.23	402.5	49.6	29.62	0.1086041E 06	19.41	0.474052E 02	2.59	50.36
10	96.9	17.33	489.7	53.0	31.64	0.1346481E 06	20.50	0.457822E 02	2.57	52.73
11	100.7	19.85	581.5	54.9	32.74	0.1762961E 06	21.94	0.434002E 02	2.54	55.71
12	104.2	21.35	678.9	58.3	34.67	0.2195964E 06	23.19	0.413188E 02	2.51	58.18
13	107.8	24.54	791.2	60.5	35.92	0.2761319E 06	24.57	0.394776E 02	2.48	60.96
14	111.5	26.63	894.7	63.0	37.37	0.3393774E 06	25.88	0.375656E 02	2.45	63.42
15	116.1	29.11	1030.3	66.4	39.29	0.4332535E 06	27.51	0.353115E 02	2.41	66.41
16	120.6	31.72	1153.9	68.2	40.30	0.5432930E 06	29.11	0.333811E 02	2.38	69.33
17	125.8	34.16	1289.1	70.8	41.71	0.6913888E 06	30.92	0.312354E 02	2.34	72.43
18	139.3	35.50	1425.2	75.3	44.08	0.1146696E 07	35.01	0.260302E 02	2.24	78.45
19	143.3	37.49	1572.5	78.7	45.99	0.1334748E 07	36.35	0.250599E 02	2.22	80.70
20	147.9	40.72	1737.3	80.0	46.72	0.1589684E 07	37.96	0.241734E 02	2.20	83.54
21	152.7	42.96	1892.4	82.6	48.16	0.1853710E 07	39.42	0.232711E 02	2.18	85.95

Table A5.9 Heat Transfer Data -- Soln.8

Run	T_s °F	ΔT °F	Q Btu/hr	$\bar{h}$ Btu/hrft²°F	$\overline{N_{Nu}}$	N_{Gr}	Z_1	N_{Pr}	Z_2	Z
1	70.9	0.70	5.1	13.7	8.29	0.1132051E 03	3.43	0.294603E 03	4.01	13.79
2	71.9	1.62	15.4	17.8	10.76	0.2900078E 03	4.39	0.275524E 03	3.95	17.33
3	73.6	3.19	34.7	20.4	12.32	0.6406201E 03	5.40	0.257776E 03	3.89	20.97
4	76.1	5.14	58.0	21.2	12.76	0.1175549E 04	6.32	0.241384E 03	3.82	24.17
5	79.0	7.39	100.2	25.4	15.32	0.1931589E 04	7.19	0.226556E 03	3.77	27.07
6	82.1	9.79	149.9	28.7	17.28	0.2930096E 04	8.01	0.213010E 03	3.71	29.72
7	85.9	12.17	194.7	30.0	18.02	0.4307277E 04	8.84	0.198118E 03	3.65	32.26
8	89.6	15.01	256.9	32.1	19.24	0.6179020E 04	9.71	0.185451E 03	3.59	34.84
9	93.4	17.83	324.2	34.1	20.41	0.8481508E 04	10.53	0.174198E 03	3.54	37.24
10	97.4	19.93	402.1	37.9	22.60	0.1128669E 05	11.33	0.161938E 03	3.47	39.37
11	102.3	23.43	499.4	40.0	23.82	0.1588312E 05	12.37	0.149931E 03	3.41	42.18
12	110.3	25.77	578.6	42.1	24.99	0.2499971E 05	13.87	0.129302E 03	3.29	45.67
13	114.6	27.94	683.1	45.9	27.15	0.3196510E 05	14.76	0.120617E 03	3.24	47.80
14	118.6	30.67	785.0	48.0	28.38	0.4004696E 05	15.64	0.113982E 03	3.19	49.94
15	123.4	33.25	896.9	50.6	29.85	0.5159359E 05	16.67	0.105978E 03	3.14	52.32
16	128.3	36.20	1020.9	52.9	31.14	0.6597188E 05	17.74	0.990000E 02	3.09	54.76
17	132.5	36.93	1075.9	54.6	32.11	0.7954819E 05	18.58	0.924022E 02	3.04	56.43
18	136.1	38.75	1155.8	55.9	32.83	0.9340356E 05	19.34	0.881405E 02	3.00	58.09
19	141.0	41.14	1283.7	58.5	34.28	0.1159448E 06	20.42	0.825680E 02	2.96	60.36
20	150.5	43.99	1442.5	61.5	35.89	0.1688751E 06	22.41	0.727037E 02	2.87	64.25
21	154.5	46.76	1579.2	63.4	36.93	0.1976613E 06	23.30	0.698230E 02	2.84	66.18
22	160.2	49.35	1731.4	65.8	38.30	0.2443178E 06	24.56	0.654240E 02	2.80	68.66
23	165.6	52.11	1883.2	67.8	39.40	0.2963523E 06	25.75	0.617857E 02	2.76	71.03

Table A5.10 Heat Transfer Data -- Soln. 9

Run	T_s °F	ΔT °F	Q Btu/hr	$\bar{h}$ Btu/hrft²°F	$\overline{N_{Nu}}$	N_{Gr}	Z_1	N_{Pr}	Z_2	Z
1	79.0	2.60	15.1	10.9	6.55	0.3395621E 02	2.58	0.104875E 04	5.31	13.71
2	81.4	5.05	34.9	13.0	7.79	0.7792960E 02	3.23	0.943169E 03	5.18	16.72
3	84.1	7.51	61.3	15.3	9.18	0.1342827E 03	3.74	0.867536E 03	5.07	18.97
4	87.6	10.74	102.2	17.9	10.70	0.2267248E 03	4.31	0.792945E 03	4.96	21.38
5	91.4	14.20	145.7	19.3	11.52	0.3530979E 03	4.86	0.728085E 03	4.86	23.60
6	95.1	17.29	195.5	21.2	12.67	0.5037590E 03	5.35	0.672560E 03	4.77	25.49
7	99.0	20.64	255.9	23.3	13.88	0.7002090E 03	5.84	0.623217E 03	4.68	27.36
8	103.4	24.29	358.8	27.7	16.51	0.9764951E 03	6.40	0.573065E 03	4.59	29.34
9	111.6	27.62	394.9	26.8	15.91	0.1613677E 04	7.34	0.480847E 03	4.39	32.24
10	115.5	30.87	488.6	29.7	17.59	0.2062909E 04	7.85	0.449544E 03	4.32	33.91
11	120.3	34.84	586.7	31.6	18.68	0.2728392E 04	8.47	0.415206E 03	4.24	35.90
12	124.7	38.18	679.0	33.4	19.69	0.3461142E 04	9.04	0.386069E 03	4.16	37.64
13	129.6	41.99	786.8	35.1	20.71	0.4453336E 04	9.69	0.356973E 03	4.09	39.56
14	134.3	45.21	895.9	37.2	21.87	0.5554723E 04	10.29	0.331810E 03	4.01	41.30
15	140.3	48.37	1026.8	39.8	23.37	0.7230070E 04	11.07	0.301311E 03	3.92	43.38
16	145.7	52.25	1160.4	41.7	24.41	0.9018418E 04	11.77	0.280224E 03	3.85	45.30
17	151.3	55.86	1286.7	43.2	25.27	0.1117116E 05	12.48	0.260250E 03	3.78	47.20
18	156.4	58.87	1440.2	45.9	26.79	0.1336200E 05	13.12	0.244231E 03	3.72	48.85
19	170.7	61.33	1570.7	48.0	27.92	0.1984309E 05	14.68	0.205395E 03	3.57	52.35

Table A5.11 Heat Transfer Data -- Soln. 10

Run	T_B °F	ΔT °F	Q Btu/hr	$\bar{h}$ Btu/hrft^2°F	$\overline{N_{Nu}}$	N_{Gr}	Z_1	N_{Pr}	Z_2	Z
1	82.5	1.72	3.9	4.2	2.54	0.2613657E 00	0.68	0.127620E 05	8.82	6.00
2	89.1	8.51	15.8	3.5	2.08	0.2328448E 01	1.27	0.827545E 04	7.97	10.15
3	94.5	14.30	34.2	4.5	2.68	0.5156793E 01	1.60	0.690047E 04	7.63	12.22
4	98.8	17.38	58.4	6.3	3.76	0.7675959E 01	1.80	0.614468E 04	7.42	13.33
5	105.6	25.06	99.4	7.4	4.43	0.1436548E 02	2.15	0.522240E 04	7.13	15.36
6	111.6	30.72	142.6	8.7	5.17	0.2208746E 02	2.44	0.457809E 04	6.91	16.86
7	117.4	34.80	194.8	10.5	6.22	0.3178757E 02	2.71	0.401132E 04	6.69	18.16
8	123.4	40.52	255.8	11.8	7.00	0.4582507E 02	3.02	0.354564E 04	6.49	19.61
9	128.9	45.10	321.0	13.4	7.88	0.6267688E 02	3.31	0.315551E 04	6.31	20.89
10	135.5	51.41	391.9	14.3	8.42	0.8919313E 02	3.67	0.277543E 04	6.12	22.47
11	142.6	56.77	488.2	16.1	9.48	0.1283577E 03	4.09	0.238983E 04	5.90	24.13
12	149.9	63.45	575.0	17.0	9.97	0.1819134E 03	4.53	0.208480E 04	5.71	25.[illegible]
13	152.1	60.47	576.4	17.9	10.46	0.2052370E 03	4.71	0.190910E 04	5.58	26.29
14	159.3	66.70	677.0	19.0	11.12	0.2840198E 03	5.19	0.167148E 04	5.40	28.06
15	166.9	72.24	779.1	20.2	11.79	0.3940071E 03	5.73	0.144767E 04	5.22	29.9[illegible]
16	174.3	78.79	876.2	20.9	12.14	0.5225625E 03	6.24	0.128829E 04	5.07	31.65
17	178.1	79.08	878.1	20.8	12.11	0.6031021E 03	6.52	0.119132E 04	4.97	32.45

Table A5.12 Heat Transfer Data -- Soln. 11

Run	T_a °F	ΔT °F	Q Btu/hr	$\overline{h}$ Btu/hrft²°F	$\overline{N_{Nu}}$	N_{Gr}	Z_1	N_{Pr}	Z_2	Z
1	75.2	1.10	15.3	26.1	15.64	0.1858107E 04	6.74	0.944056E 02	3.09	20.85
2	77.0	2.52	34.8	25.9	15.53	0.4654941E 04	8.51	0.906724E 02	3.06	26.04
3	78.5	3.68	59.6	30.4	18.19	0.7263164E 04	9.52	0.882332E 02	3.04	28.95
4	81.9	5.76	101.2	32.9	19.68	0.1330626E 05	11.09	0.830657E 02	3.00	33.22
5	83.9	7.77	146.4	35.4	21.09	0.1935099E 05	12.18	0.806071E 02	2.97	36.24
6	86.4	9.65	193.7	37.7	22.43	0.2648658E 05	13.18	0.776506E 02	2.95	38.86
7	89.0	11.78	256.3	40.8	24.28	0.3560296E 05	14.20	0.748117E 02	2.92	41.48
8	91.4	13.67	321.9	44.2	26.23	0.4537935E 05	15.09	0.721530E 02	2.90	43.69
9	94.5	16.24	405.1	46.8	27.74	0.6036007E 05	16.21	0.690490E 02	2.86	46.42
10	97.4	18.41	487.0	49.6	29.36	0.7629013E 05	17.18	0.661796E 02	2.83	48.70
11	100.7	20.85	580.9	52.3	30.85	0.9825956E 05	18.30	0.629168E 02	2.80	51.23
12	105.2	23.00	679.4	55.4	32.60	0.1327906E 06	19.71	0.580657E 02	2.75	54.12
13	108.7	25.25	777.8	57.8	33.91	0.1681738E 06	20.90	0.548147E 02	2.71	56.57
14	112.5	27.63	892.0	60.6	35.45	0.2144399E 06	22.20	0.515036E 02	2.67	59.16
15	116.7	30.51	1023.2	62.9	36.72	0.2787803E 06	23.68	0.481515E 02	2.62	62.09
16	120.5	32.81	1150.7	65.8	38.30	0.3499571E 06	25.05	0.451917E 02	2.58	64.64
17	124.5	35.25	1258.5	67.0	38.87	0.4415981E 06	26.52	0.422047E 02	2.54	67.32
18	128.7	37.12	1420.6	71.8	41.54	0.5585521E 06	28.09	0.390595E 02	2.49	69.96
19	141.3	38.50	1588.6	77.4	44.25	0.1096891E 07	33.07	0.297117E 02	2.33	77.01
20	145.6	40.25	1731.8	80.7	45.99	0.1360003E 07	34.84	0.275966E 02	2.29	79.67
21	151.0	42.41	1888.3	83.5	47.38	0.1744001E 07	36.99	0.253513E 02	2.24	82.84
22	156.4	44.36	2027.1	85.7	48.43	0.2168718E 07	38.97	0.235417E 02	2.20	85.70

Table A5.13 Heat Transfer Data -- Soln. 12

Run	T_B °F	ΔT °F	Q Btu/hr	$\bar{h}$ Btu/hrft^2°F	$\overline{N_{Nu}}$	N_{Gr}	Z_1	N_{Pr}	Z_2	Z
1	80.3	2.60	15.9	11.5	6.87	0.3555459E 01	1.38	0.366778E 04	7.64	10.55
2	82.8	4.97	34.6	13.1	7.79	0.7698755E 01	1.68	0.345532E 04	7.53	12.65
3	86.3	8.35	62.3	14.0	8.33	0.1515254E 02	2.00	0.321347E 04	7.39	14.76
4	90.5	11.44	99.5	16.3	9.68	0.2562024E 02	2.28	0.292852E 04	7.22	16.49
5	94.1	14.90	143.9	18.1	10.74	0.3886259E 02	2.54	0.273497E 04	7.10	18.03
6	98.1	18.70	195.6	19.6	11.60	0.5739101E 02	2.80	0.254266E 04	6.97	19.55
7	102.7	22.64	254.9	21.1	12.46	0.8510741E 02	3.10	0.232242E 04	6.82	21.14
8	107.2	26.72	326.6	22.9	13.49	0.1210383E 03	3.39	0.213546E 04	6.68	22.64
9	111.9	30.60	404.7	24.8	14.55	0.1695927E 03	3.70	0.194973E 04	6.53	24.13
10	116.3	34.06	485.7	26.8	15.65	0.2293538E 03	3.99	0.178501E 04	6.39	25.49
11	122.1	37.90	583.5	28.9	16.83	0.3365164E 03	4.40	0.157395E 04	6.19	27.24
12	126.5	41.21	676.0	30.8	17.88	0.4404055E 03	4.71	0.144554E 04	6.06	29.57
13	132.6	46.19	785.5	31.9	18.47	0.6306018E 03	5.17	0.129082E 04	5.89	30.44
14	136.9	49.03	881.4	33.7	19.46	0.9050151E 03	5.50	0.118503E 04	5.77	31.72
15	148.6	50.54	1017.9	37.8	21.57	0.1585702E 04	6.54	0.876666E 03	5.35	35.00
16	153.8	55.03	1157.6	39.5	22.47	0.2045370E 04	6.98	0.808742E 03	5.25	36.61
17	158.4	58.36	1282.7	41.2	23.41	0.2558053E 04	7.39	0.747566E 03	5.14	38.02
18	163.2	62.42	1423.2	42.8	24.21	0.3188489E 04	7.82	0.694534E 03	5.05	39.49
19	169.6	65.12	1585.1	45.7	25.73	0.4431367E 04	8.51	0.604868E 03	4.88	41.52
20	174.0	67.41	1705.9	47.5	26.66	0.5597797E 04	9.03	0.548913E 03	4.76	43.02

Table A5.14 Heat Transfer Data -- Soln. 13

Run	T_B °F	ΔT °F	Q Btu/hr	$\overline{h}$ Btu/hrft^2°F	$\overline{N_{Nu}}$	N_{Gr}	Z_1	N_{Pr}	Z_2	Z
1	76.2	1.49	4.0	5.0	3.01	0.1980977E-01	0.36	0.403478E 05	13.19	4.70
2	79.2	4.79	15.4	6.0	3.62	0.7806224E-01	0.51	0.355410E 05	12.79	6.54
3	83.1	8.53	34.8	7.6	4.57	0.1682243E 00	0.63	0.321687E 05	12.48	7.81
4	88.8	13.93	60.4	8.1	4.84	0.3497347E 00	0.76	0.285690E 05	12.12	9.19
5	94.0	18.04	101.7	10.6	6.27	0.5683427E 00	0.86	0.257003E 05	11.81	10.17
6	99.4	22.82	146.0	12.0	7.10	0.8955235E 00	0.97	0.231733E 05	11.51	11.18
7	105.3	27.35	195.4	13.4	7.90	0.1391133E 01	1.09	0.205379E 05	11.17	12.18
8	112.7	32.64	250.6	14.4	8.45	0.2323336E 01	1.25	0.175471E 05	10.74	13.41
9	119.1	38.44	327.5	16.0	9.34	0.3513775E 01	1.39	0.155592E 05	10.42	14.52
10	126.4	44.49	405.6	17.1	9.95	0.5507500E 01	1.57	0.134460E 05	10.06	15.78
11	132.3	49.43	488.5	18.5	10.75	0.7860057E 01	1.72	0.119082E 05	9.76	16.82
12	138.5	54.50	582.0	20.0	11.58	0.1120572E 02	1.89	0.105034E 05	9.46	17.91
13	145.4	60.45	683.5	21.2	12.21	0.1644095E 02	2.10	0.915147E 04	9.14	19.17
14	152.9	65.36	784.3	22.5	12.89	0.2525433E 02	2.35	0.769992E 04	8.76	20.58
15	159.1	69.39	886.3	24.0	13.67	0.3533293E 02	2.57	0.671688E 04	8.47	21.76
16	167.0	75.18	1021.2	25.5	14.46	0.5232158E 02	2.85	0.574876E 04	8.15	23.25
17	180.9	80.57	1134.4	26.4	14.83	0.9439261E 02	3.34	0.444056E 04	7.64	25.56

Table A5.15 Heat Transfer Data -- Soln. 1

Run	T_s °F	ΔT °F	Q Btu/hr	$\overline{h}$ Btu/hrft²°F	$\overline{N_{Nu}}$	N_{Gr}	N_{Pr}'	$N_{Gr}N_{Pr}'$
1	73.7	0.49	4.7	18.1	10.85	0.2544422E 04	0.8849768E 02	0.2251754E 06
2	74.5	1.12	15.6	26.2	15.66	0.6605121E 04	0.8823947E 02	0.5828324E 06
3	76.0	2.11	34.7	30.8	18.41	0.1410238E 05	0.8765993E 02	0.1236213E 07
4	77.4	3.43	60.7	33.2	19.81	0.2527158E 05	0.8719637E 02	0.2203590E 07
5	79.6	4.84	101.4	39.3	23.45	0.3952333E 05	0.8639420E 02	0.3414586E 07
6	81.3	6.20	145.0	43.9	26.15	0.5457687E 05	0.8579970E 02	0.4682679E 07
7	83.6	8.13	194.2	44.8	26.67	0.7824091E 05	0.8504782E 02	0.6654210E 07
8	87.8	9.91	256.1	48.5	28.78	0.1109918E 06	0.8324971E 02	0.9240036E 07
9	90.3	11.94	329.7	51.8	30.72	0.1444940E 06	0.8249274E 02	0.1191971E 08
10	93.0	14.13	402.1	53.4	31.62	0.1855433E 06	0.8163701E 02	0.1514720E 08
11	96.2	16.02	489.8	57.4	33.91	0.2314270E 06	0.8048973E 02	0.1862749E 08
12	99.6	18.18	587.5	60.6	35.78	0.2890132E 06	0.7933656E 02	0.2292931E 08
13	102.9	20.25	679.6	62.9	37.09	0.3521072E 06	0.7823357E 02	0.2754659E 08
14	106.6	22.57	783.6	65.1	38.30	0.4337464E 06	0.7697736E 02	0.3338864E 08
15	110.9	24.59	891.3	68.0	39.89	0.5282866E 06	0.7550658E 02	0.3988910E 08
16	116.3	27.29	1029.0	70.7	41.38	0.6718904E 06	0.7371574E 02	0.4952890E 08
17	127.4	28.35	1150.9	76.1	44.22	0.9350886E 06	0.6954971E 02	0.6503514E 08
18	131.2	31.53	1283.7	76.4	44.28	0.1117933E 07	0.6875188E 02	0.7685998E 08
19	135.8	33.69	1432.9	79.8	46.16	0.1314000E 07	0.6756311E 02	0.8877792E 08
20	139.4	35.31	1574.8	83.7	48.32	0.1479599E 07	0.6668198E 02	0.9866258E 08
21	146.5	38.19	1715.7	84.3	48.51	0.1832543E 07	0.6505188E 02	0.1192104E 09
22	151.9	39.52	1812.5	86.0	49.39	0.2099366E 07	0.6384383E 02	0.1340316E 09
23	157.3	38.79	1789.1	86.5	49.53	0.2279442E 07	0.6262634E 02	0.1427531E 09
24	160.5	40.86	1917.7	88.0	50.35	0.2513268E 07	0.6221895E 02	0.1563729E 09

Table A5.16 Heat Transfer Data -- Soln. 2

Run	T_s °F	ΔT °F	Q Btu/hr	$\overline{h}$ Btu/hrft²°F	$\overline{N_{Nu}}$	N_{Gr}	N_{Pr}'	$N_{Gr}N_{Pr}'$
1	82.0	4.02	46.9	21.9	13.02	0.5045642E 03	0.6735684E 03	0.3398584E 06
2	84.2	6.11	81.4	25.0	14.85	0.8244863E 03	0.6873965E 03	0.5667490E 06
3	86.4	8.34	121.5	27.3	16.20	0.1189337E 04	0.7006638E 03	0.8333253E 06
4	89.5	10.73	168.5	29.4	17.47	0.1575760E 04	0.7231758E 03	0.1139551E 07
5	92.5	13.39	227.6	31.9	18.89	0.2029939E 04	0.7424045E 03	0.1507035E 07
6	96.7	16.29	293.8	33.8	20.00	0.2518035E 04	0.7734585E 03	0.1947595E 07
7	100.0	19.23	361.8	35.3	20.83	0.3044779E 04	0.7938704E 03	0.2417159E 07
8	104.0	21.49	444.3	38.8	22.84	0.3421687E 04	0.8268811E 03	0.2829328E 07
9	108.5	25.86	536.8	38.9	22.90	0.4248383E 04	0.8522844E 03	0.3620830E 07
10	116.2	28.64	629.5	41.2	24.13	0.4684215E 04	0.9213240E 03	0.4315679E 07
11	120.9	31.94	727.1	42.7	24.94	0.5313199E 04	0.9551135E 03	0.5074708E 07
12	126.3	36.47	845.9	43.5	25.36	0.6227547E 04	0.9884038E 03	0.6155331E 07
13	131.1	39.97	959.5	45.0	26.19	0.6943973E 04	0.1022029E 04	0.7096940E 07
14	136.5	43.80	1081.3	46.3	26.87	0.7751133E 04	0.1059109E 04	0.8209297E 07
15	144.7	47.33	1218.7	48.3	27.91	0.8468020E 04	0.1129219E 04	0.9562244E 07
16	151.1	51.04	1348.2	49.6	28.55	0.9273988E 04	0.1179385E 04	0.1093760E 08
17	155.8	49.67	1285.0	48.5	27.88	0.8859777E 04	0.1241204E 04	0.1099679E 08
18	161.2	53.74	1429.3	49.9	28.61	0.9768805E 04	0.1280174E 04	0.1250577E 08
19	166.2	57.27	1578.7	51.7	29.61	0.1056049E 05	0.1318708E 04	0.1392621E 08

Table A5.17 Heat Transfer Data -- Soln. 3

Run	T_s °F	ΔT °F	Q Btu/hr	$\bar{h}$ Btu/hrft²°F	$\overline{N_{Nr}}$	N_{Gr}	N_{Pr}	$N_{Gr}N_{Pr}$
1	76.5	1.11	3.7	6.3	3.78	0.1156423E 01	0.4230750E 04	0.4892531E 04
2	78.1	3.06	15.2	9.3	5.54	0.5063620E 01	0.4294169E 04	0.2174403E 05
3	82.3	7.14	34.7	9.1	5.43	0.1634015E 02	0.4539512E 04	0.7417625E 05
4	86.4	10.71	60.9	10.7	6.34	0.2772763E 02	0.4788941E 04	0.1327859E 06
5	92.4	16.27	101.6	11.7	6.95	0.4760570E 02	0.5120930E 04	0.2437854E 06
6	97.6	21.36	147.0	12.9	7.64	0.6750116E 02	0.5394629E 04	0.3641437E 06
7	104.3	26.88	195.3	13.6	8.04	0.8868904E 02	0.5772246E 04	0.5119349E 06
8	110.9	32.56	257.1	14.8	8.71	0.1119423E 03	0.6120039E 04	0.6850912E 06
9	118.5	39.52	329.5	15.6	9.17	0.1429366E 03	0.6482844E 04	0.9266354E 06
10	128.6	46.03	403.2	16.4	9.59	0.1679998E 03	0.7034551E 04	0.1181803E 07
11	136.9	53.37	490.8	17.3	10.04	0.2038352E 03	0.7387059E 04	0.1505742E 07
12	146.0	61.00	584.7	18.0	10.43	0.2425326E 03	0.7758688E 04	0.1881734E 07
13	155.4	68.90	679.9	18.5	10.70	0.2850291E 03	0.8110441E 04	0.2311711E 07
14	165.9	77.53	784.1	19.0	10.92	0.3335376E 03	0.8471652E 04	0.2825614E 07
15	172.8	81.89	886.2	20.3	11.66	0.3551709E 03	0.8730730E 04	0.3100901E 07
16	183.5	89.55	1015.4	21.3	12.17	0.3969822E 03	0.9067816E 04	0.3599761E 07

Table A5.18 Heat Transfer Data -- Soln. 4

Run	T_B °F	ΔT °F	Q Btu/hr	$\overline{h}$ Btu/hrft²°F	$\overline{N_{Nu}}$	N_{Gr}	N_{Pr}'	$N_{Gr}N_{Pr}'$
1	79.3	1.74	31.4	33.8	20.32	0.6317129E 05	0.2366432E 02	0.1494905E 07
2	77.2	2.08	34.9	31.4	18.93	0.6601006E 05	0.2454401E 02	0.1620151E 07
3	79.9	4.76	98.4	38.8	23.32	0.1654518E 06	0.2401500E 02	0.3973325E 07
4	84.1	7.77	189.9	45.8	27.51	0.3176579E 06	0.2304181E 02	0.7319412E 07
5	88.9	10.40	284.0	51.2	30.67	0.5160993E 06	0.2192021E 02	0.1131301E 08
6	94.4	13.14	394.7	56.3	33.64	0.8042532E 06	0.2078171E 02	0.1671375E 08
7	99.2	15.01	493.4	61.7	36.72	0.1100597E 07	0.1987009E 02	0.2187094E 08
8	103.6	17.00	583.1	64.3	38.23	0.1447649E 07	0.1917729E 02	0.2776198E 08
9	99.0	20.85	747.6	67.3	40.13	0.1336817E 07	0.2058102E 02	0.2751306E 08
10	110.3	23.27	878.0	70.8	41.96	0.2283206E 07	0.1859819E 02	0.4246349E 08
11	123.6	26.75	1075.8	75.4	44.42	0.3762732E 07	0.1740402E 02	0.6548666E 08
12	127.9	27.05	1139.5	79.0	46.42	0.4178520E 07	0.1721523E 02	0.7193416E 08
13	134.0	29.14	1313.6	84.6	49.54	0.4977054E 07	0.1711070E 02	0.8516088E 08
14	141.1	31.12	1418.6	85.5	49.96	0.5847640E 07	0.1713684E 02	0.1002101E 09
15	145.8	33.22	1581.6	89.3	52.10	0.6564910E 07	0.1721025E 02	0.1129837E 09
16	151.8	35.47	1724.5	91.2	53.10	0.7433239E 07	0.1734445E 02	0.1289255E 09
17	159.0	37.27	1896.7	95.5	55.48	0.8377788E 07	0.1753204E 02	0.1468797E 09
18	157.2	35.43	1769.6	93.7	54.46	0.7872197E 07	0.1750833E 02	0.1378290E 09
19	147.7	29.84	1378.4	86.6	50.48	0.6115418E 07	0.1730556E 02	0.1058307E 09
20	115.7	14.21	492.9	65.1	38.35	0.1879950E 07	0.1751048E 02	0.3291983E 08
21	97.3	10.02	283.2	53.0	31.56	0.7499202E 06	0.1973045E 02	0.1479627E 08
22	87.7	4.53	102.3	42.4	25.33	0.2427214E 06	0.2141086E 02	0.5196874E 07
23	79.4	0.91	14.3	29.5	17.73	0.3369720E 05	0.2348958E 02	0.7915331E 06
24	79.6	2.05	34.1	31.2	18.75	0.7523594E 05	0.2360826E 02	0.1776189E 07

Table A5.19 Heat Transfer Data -- Soln. 5

Run	T_B °F	T °F	Q Btu/hr	$\bar{h}$ Btu/hrft²°F	N_{Nu}	N_{Gr}	N_{Pr}	$N_{Gr}N_{Pr}$
1	76.3	0.88	8.3	17.7	10.65	0.2206246E 04	0.1139065E 03	0.2513058E 06
2	76.3	1.26	12.9	19.3	11.60	0.3221057E 04	0.1142497E 03	0.3680047E 06
3	75.1	1.87	23.3	23.4	14.15	0.4609273E 04	0.1162830E 03	0.5359799E 06
4	76.1	2.71	37.5	26.0	15.64	0.7105559E 04	0.1154371E 03	0.8202454E 06
5	77.4	3.44	52.8	28.8	17.32	0.9593207E 04	0.1142772E 03	0.1096284E 07
6	79.2	5.44	98.7	34.1	20.49	0.1644568E 05	0.1131034E 03	0.1862324E 07
7	81.3	8.89	140.1	29.6	17.73	0.2865302E 05	0.1125578E 03	0.3225119E 07
8	88.0	8.84	193.5	41.1	24.59	0.3873356E 05	0.1042346E 03	0.4037899E 07
9	90.4	10.80	254.3	44.2	26.42	0.5148447E 05	0.1025191E 03	0.5278143E 07
10	97.1	14.40	377.5	49.2	29.31	0.8670888E 05	0.9730061E 02	0.8436826E 07
11	99.4	15.51	425.0	51.4	30.61	0.1005346E 06	0.9567549E 02	0.9618699E 07
12	102.1	17.12	489.0	53.6	31.87	0.1209009E 06	0.9385078E 02	0.1134665E 08
13	107.6	18.69	555.7	55.8	33.07	0.1595667E 06	0.9966484E 02	0.1430752E 08
14	110.0	19.51	601.8	57.9	34.27	0.1797508E 06	0.8806314E 02	0.1582942E 08
15	113.3	20.46	644.6	59.1	34.94	0.2087074E 06	0.8595601E 02	0.1793965E 08
16	115.0	21.40	693.3	60.8	35.90	0.2287564E 06	0.8505884E 02	0.1945774E 08
17	116.4	22.30	729.1	61.3	36.21	0.2472115E 06	0.8438365E 02	0.2086061E 08
18	118.2	23.36	776.2	62.3	36.77	0.2722008E 06	0.8344951E 02	0.2271502E 08
19	121.0	24.93	846.6	53.7	37.54	0.3123878E 06	0.8210881E 02	0.2564978E 08
20	125.2	26.48	936.6	66.4	39.03	0.3715778E 06	0.7999770E 02	0.2972536E 08
21	128.9	28.12	1032.7	68.9	40.46	0.4336313E 06	0.7831636E 02	0.3396042E 08
22	132.8	29.90	1124.2	70.5	41.36	0.5070571E 06	0.7667474E 02	0.3887846E 08
23	137.4	32.68	1277.4	73.3	42.93	0.6132456E 06	0.7502467E 02	0.4600854E 08
24	142.3	34.35	1373.6	75.0	43.83	0.7263646E 06	0.7303186E 02	0.5304774E 08
25	146.6	36.72	1517.7	77.5	45.24	0.8523254E 06	0.7157048E 02	0.6100133E 08
26	151.7	38.28	1627.9	79.8	46.47	0.1000440E 07	0.6966469E 02	0.6969536E 08
27	155.8	40.19	1753.0	81.8	47.62	0.1149306E 07	0.6826988E 02	0.7846298E 08
28	160.5	42.14	1890.2	84.1	48.91	0.1340471E 07	0.6660849E 02	0.8928674E 08

Table A5.20 Heat Transfer Data -- Soln. 6

Run	T_B °F	ΔT °F	Q Btu/hr	$\overline{h}$ Btu/hrft²°F	$\overline{N_{Nu}}$	N_{Gr}	N_{Pr}'	$N_{Gr}N_{Pr}$'
1	73.8	0.62	4.9	14.7	8.85	0.8413762E 02	0.4207239E 03	0.3539870E 05
2	73.9	0.72	4.9	12.6	7.62	0.9925151E 02	0.4202888E 03	0.4171430E 05
3	75.6	1.90	15.5	15.3	9.24	0.3024688E 03	0.4122280E 03	0.1246861E 06
4	76.9	2.91	26.3	17.0	10.21	0.5024504E 03	0.4065024E 03	0.2042473E 06
5	79.1	4.45	41.2	17.4	10.44	0.8614985E 03	0.3967537E 03	0.3418027E 06
6	80.6	5.48	57.6	19.7	11.85	0.1145361E 04	0.3895386E 03	0.4461624E 06
7	82.7	7.07	86.4	22.9	13.77	0.1619167E 04	0.3810696E 03	0.6170153E 06
8	83.9	7.97	85.2	20.1	12.04	0.1925129E 04	0.3759019E 03	0.7236594E 06
9	84.9	8.42	108.8	24.2	14.54	0.2131541E 04	0.3709865E 03	0.7905596E 06
10	85.9	9.38	130.5	26.1	15.65	0.2473527E 04	0.3671333E 03	0.9081141E 06
11	88.3	10.22	153.9	28.3	16.92	0.3025566E 04	0.3545156E 03	0.1072610E 07
12	90.8	11.80	185.7	29.5	17.64	0.3885597E 04	0.3437319E 03	0.1335603E 07
13	92.8	13.17	219.6	31.3	18.69	0.4714711E 04	0.3353743E 03	0.1581192E 07
14	95.8	15.22	271.4	33.4	19.97	0.6116016E 04	0.3239639E 03	0.1981368E 07
15	98.9	17.43	331.2	35.6	21.24	0.7923262E 04	0.3118997E 03	0.2471262E 07
16	102.2	18.95	378.6	37.5	22.30	0.9945051E 04	0.2975903E 03	0.2959550E 07
17	105.5	20.94	439.3	39.4	23.38	0.1250520E 05	0.2853254E 03	0.3568052E 07
18	109.5	22.36	491.3	41.2	24.44	0.1595743E 05	0.2684453E 03	0.4283698E 07
19	113.4	24.80	585.3	44.3	26.21	0.2056587E 05	0.2551997E 03	0.5248402E 07
20	117.0	26.41	652.0	46.3	27.37	0.2547904E 05	0.2420689E 03	0.6167682E 07
21	121.4	28.53	729.9	48.0	28.30	0.3276197E 05	0.2276828E 03	0.7459337E 07
22	124.7	30.36	805.4	49.8	29.31	0.3942229E 05	0.2180315E 03	0.8595299E 07
23	130.4	31.87	884.0	52.0	30.56	0.5282189E 05	0.1994512E 03	0.1053539E 08
24	134.9	34.73	1002.9	54.2	31.77	0.6677588E 05	0.1892255E 03	0.1263570E 08
25	141.3	36.72	1119.7	57.2	33.46	0.8963663E 05	0.1733898E 03	0.1554208E 08
26	147.8	39.87	1267.2	59.6	34.80	0.1190536E 06	0.1613182E 03	0.1920550E 08
27	151.1	41.69	1364.6	61.4	35.80	0.1359918E 06	0.1563913E 03	0.2126792E 09
28	155.5	43.24	1478.3	64.1	37.34	0.1590231E 06	0.1499311E 03	0.2384250E 08
29	161.4	45.70	1574.1	64.6	37.57	0.1912333E 06	0.1435988E 03	0.2746088E 09
30	164.7	47.99	1663.2	65.0	37.78	0.2113823E 06	0.1414127E 03	0.2989214E 09
31	165.0	47.54	1670.4	65.9	38.30	0.2112858E 06	0.1409745E 03	0.2978590E 08

Table A5.21 Heat Transfer Data -- Soln.7

Run	T_s °F	ΔT °F	Q Btu/hr	$\bar{h}$ Btu/hrft^2°F	$\overline{N_{Nu}}$	N_{Gr}	N_{Pr}'	$N_{Gr}' N_{Pr}'$
1	73.5	0.45	4.4	18.5	11.16	0.1318661E 04	0.9055602E 02	0.1194259E 06
2	75.8	2.52	33.5	25.0	15.05	0.8619379E 04	0.8832072E 02	0.7612698E 06
3	77.9	3.91	60.7	29.1	17.54	0.1477194E 05	0.8591159E 02	0.1269072E 07
4	79.8	5.56	100.2	33.8	20.33	0.2267509E 05	0.8410229E 02	0.1907027E 07
5	82.1	7.19	145.6	38.0	22.83	0.3220632E 05	0.8166948E 02	0.2630273E 07
6	85.6	8.75	197.0	42.2	25.33	0.4569592E 05	0.7748460E 02	0.3540730E 07
7	88.3	11.06	256.6	43.5	26.07	0.6351314E 05	0.7511284E 02	0.4770652E 07
8	91.3	13.17	340.4	48.5	29.00	0.8461600E 05	0.7224352E 02	0.6112957E 07
9	94.3	15.23	402.5	49.6	29.62	0.1086041E 06	0.6962790E 02	0.7561877E 07
10	96.9	17.33	489.7	53.0	31.64	0.1346481E 06	0.6753790E 02	0.9093851E 07
11	100.7	19.85	581.5	54.9	32.74	0.1762961E 06	0.6429820E 02	0.1133552E 08
12	104.2	21.85	678.9	58.3	34.67	0.2195964E 06	0.6137741E 02	0.1347826E 08
13	107.8	24.54	791.2	60.5	35.92	0.2761319E 06	0.5882709E 02	0.1624403E 08
14	111.5	26.63	894.7	63.0	37.37	0.3393774E 06	0.5606885E 02	0.1902850E 08
15	116.1	29.11	1030.3	66.4	39.29	0.4332535E 06	0.5276607E 02	0.2286107E 08
16	120.6	31.72	1153.8	69.2	40.30	0.5432930E 06	0.4992009E 02	0.2712123E 08
17	125.8	34.16	1289.1	70.8	41.71	0.6913888E 06	0.4668285E 02	0.3227598E 08
18	139.3	35.50	1425.2	75.3	44.08	0.1146696E 07	0.3845662E 02	0.4409805E 08
19	143.3	37.49	1572.5	78.7	45.99	0.1334748E 07	0.3694431E 02	0.4931133E 08
20	147.9	40.72	1737.3	80.0	46.72	0.1589684E 07	0.3557922E 02	0.5655971E 08
21	152.7	42.96	1892.4	82.6	48.16	0.1853710E 07	0.3412898E 02	0.6326523E 08

Table A5.22 Heat Transfer Data --. Soln. 8

Run	T_B °F	ΔT °F	Q Btu/hr	$\bar{h}$ Btu/hrft²°F	$\overline{N_{Nu}}$	N_{Gr}	N_{Pr}	$N_{Gr}N_{Pr}$
1	70.9	0.70	5.1	13.7	8.29	0.1132051E 03	0.3920066E 03	0.4437714E 05
2	71.9	1.62	15.4	17.8	10.76	0.2900078E 03	0.3877947E 03	0.1124634E 06
3	73.6	3.19	34.7	20.4	12.32	0.6406201E 03	0.3800266E 03	0.2434527E 06
4	76.1	5.14	58.0	21.2	12.76	0.1175549E 04	0.3681736E 03	0.4328060E 06
5	79.0	7.39	100.2	25.4	15.32	0.1931589E 04	0.3548782E 03	0.6854787E 06
6	82.1	9.79	149.9	28.7	17.28	0.2930096E 04	0.3409027E 03	0.9985847E 06
7	85.9	12.17	194.7	30.0	18.02	0.4307277E 04	0.3226235E 03	0.1389629E 07
8	89.6	15.01	256.9	32.1	19.24	0.6179020E 04	0.3069441E 03	0.1896613E 07
9	93.4	17.83	324.2	34.1	20.41	0.8481508E 04	0.2922004E 03	0.2478300E 07
10	97.4	19.93	402.1	37.9	22.60	0.1128669E 05	0.2743098E 03	0.3096050E 07
11	102.3	23.43	499.4	40.0	23.82	0.1588312E 05	0.2571113E 03	0.4083729E 07
12	110.3	25.77	578.6	42.1	24.99	0.2499971E 05	0.2237958E 03	0.5594832E 07
13	114.6	27.94	683.1	45.9	27.15	0.3196510E 05	0.2099246E 03	0.6710262E 07
14	118.6	30.67	785.0	48.0	28.38	0.4004696E 05	0.1994933E 03	0.7989103E 07
15	123.4	33.25	896.9	50.6	29.85	0.5159359E 05	0.1862912E 03	0.9611432E 07
16	128.3	36.20	1020.9	52.9	31.14	0.6597188E 05	0.1747021E 03	0.1152543E 08
17	132.5	36.93	1075.9	54.6	32.11	0.7954819E 05	0.1629767E 03	0.1296450E 08
18	136.1	38.75	1155.8	55.9	32.83	0.9340356E 05	0.1556296E 03	0.1453636E 08
19	141.0	41.14	1283.7	58.5	34.28	0.1159448E 06	0.1458541E 03	0.1691101E 08
20	150.5	43.99	1442.5	61.5	35.89	0.1688751E 06	0.1278646E 03	0.2159314E 08
21	154.5	46.76	1579.2	63.4	36.93	0.1976613E 06	0.1227963E 03	0.2427206E 08
22	160.2	49.35	1731.4	65.8	38.30	0.2443178E 06	0.1146953E 03	0.2802208E 08
23	165.6	52.11	1883.2	67.8	39.40	0.2963523E 06	0.1079799E 03	0.3200008E 08

Table A5.23 Heat Transfer Data -- Soln. 9

Run	T_s °F	ΔT °F	Q Btu/hr	$\overline{h}$ Btu/hrft²°F	$\overline{N_{Nu}}$	N_{Gr}	N_{Pr}	$N_{Gr}N_{Pr}$
1	79.0	2.60	15.1	10.9	6.55	0.3395621E 02	0.1492984E 04	0.5069608E 05
2	81.4	5.05	34.9	13.0	7.79	0.7792960E 02	0.1460539E 04	0.1138192E 06
3	84.1	7.51	61.3	15.3	9.18	0.1342827E 03	0.1420692E 04	0.1907743E 06
4	87.6	10.74	102.2	17.9	10.70	0.2267248E 03	0.1371357E 04	0.3109206E 06
5	91.4	14.20	145.7	19.3	11.52	0.3530979E 03	0.1319658E 04	0.4659686E 06
6	95.1	17.29	195.5	21.2	12.67	0.5037590E 03	0.1266844E 04	0.6381838E 06
7	99.0	20.64	255.9	23.3	13.88	0.7002090E 03	0.1217021E 04	0.8521692E 06
8	103.4	24.29	358.8	27.7	16.51	0.9764951E 03	0.1161439E 04	0.1134139E 07
9	111.6	27.62	394.9	26.8	15.91	0.1613677E 04	0.1034994E 04	0.1670146E 07
10	115.5	30.87	488.6	29.7	17.59	0.2062909E 04	0.9956843E 03	0.2054005E 07
11	120.3	34.84	586.7	31.6	18.68	0.2728392E 04	0.9505652E 03	0.2593514E 07
12	124.7	38.18	679.0	33.4	19.69	0.3461142E 04	0.9097585E 03	0.3148803E 07
13	129.6	41.99	786.8	35.1	20.71	0.4453336E 04	0.8677649E 03	0.3864448E 07
14	134.3	45.21	895.9	37.2	21.87	0.5554723E 04	0.8295515E 03	0.4607928E 07
15	140.3	48.37	1026.8	39.8	23.37	0.7230070E 04	0.7801768E 03	0.5640732E 07
16	145.7	52.25	1160.4	41.7	24.41	0.9018418E 04	0.7470662E 03	0.6737354E 07
17	151.3	55.86	1286.7	43.2	25.27	0.1117116E 05	0.7144971E 03	0.7981761E 07
18	156.4	58.87	1440.2	45.9	26.79	0.1336200E 05	0.6878179E 03	0.9190625E 07
19	170.7	61.33	1570.7	48.0	27.92	0.1984309E 05	0.6180110E 03	0.1226325E 08

Table A5.24 Heat Transfer Data -- Soln. 10

Run	T_s °F	ΔT °F	Q Btu/hr	$\bar{h}$ Btu/hrft^2°F	$\overline{N_{Nu}}$	N_{Gr}	N_{Pr}'	$N_{Gr}N_{Pr}'$
1	82.5	1.72	3.9	4.2	2.54	0.2613657E 00	0.1009207E 05	0.2637721E 04
2	89.1	8.51	15.8	3.5	2.08	0.2328448E 01	0.9605359E 04	0.2236558E 05
3	94.5	14.30	34.2	4.5	2.63	0.5156793E 01	0.9231309E 04	0.4760394E 05
4	98.8	17.38	58.4	6.3	3.76	0.7675959E 01	0.8842395E 04	0.6787381E 05
5	105.6	25.06	99.4	7.4	4.43	0.1436548E 02	0.8433879E 04	0.1211567E 06
6	111.6	30.72	142.6	8.7	5.17	0.2208746E 02	0.8021309E 04	0.1771703E 06
7	117.4	34.80	194.8	10.5	6.22	0.3178757E 02	0.7549395E 04	0.2399769E 06
8	123.4	40.52	255.8	11.8	7.00	0.4582507E 02	0.7171367E 04	0.3286284E 06
9	128.9	45.10	321.0	13.4	7.88	0.6267688E 02	0.6800320E 04	0.4262228E 06
10	135.5	51.41	391.9	14.3	8.42	0.8919313E 02	0.6428441E 04	0.5733728E 06
11	142.6	56.77	488.2	16.1	9.48	0.1283577E 03	0.5980605E 04	0.7676567E 06
12	149.9	63.45	575.0	17.0	9.97	0.1815134E 03	0.5619207E 04	0.1019961E 07
13	152.1	60.47	576.4	17.9	10.46	0.2052370E 03	0.5311473E 04	0.1090110E 07
14	159.3	66.70	677.0	19.0	11.12	0.2840198E 03	0.4999176E 04	0.1419864E 07
15	166.9	72.24	779.1	20.2	11.79	0.3940071E 03	0.4671934E 04	0.1840774E 07
16	174.3	78.79	876.2	20.9	12.14	0.5225625E 03	0.4442359E 04	0.2321410E 07
17	178.1	79.08	878.1	20.8	12.11	0.6031021E 03	0.4268785E 04	0.2574513E 07

Table A5.25 Heat Transfer Data -- Soln. 11

Run	T_B °F	ΔT °F	Q Btu/hr	$\overline{h}$ Btu/hrft²°F	$\overline{N_{Nu}}$	N_{Gr}	N_{Pr}'	$N_{Gr}N_{Pr}'$
1	75.2	1.10	15.3	26.1	15.64	0.1858107E 04	0.1101950E 03	0.2047540E 06
2	77.0	2.52	34.8	25.9	15.53	0.4654941E 04	0.1076267E 03	0.5009960E 06
3	78.5	3.68	59.6	30.4	18.19	0.7263164E 04	0.1054832E 03	0.7661419E 06
4	81.9	5.76	101.2	32.9	19.68	0.1330626E 05	0.9999194E 02	0.1330519E 07
5	83.9	7.77	146.4	35.4	21.09	0.1935099E 05	0.9749416E 02	0.1886608E 07
6	86.4	9.65	193.7	37.7	22.43	0.2648658E 05	0.9414604E 02	0.2493606E 07
7	89.0	11.78	256.3	40.8	24.28	0.3560286E 05	0.9087170E 02	0.3235292E 07
8	91.4	13.67	321.9	44.2	26.23	0.4537935E 05	0.8770296E 02	0.3979903E 07
9	94.5	16.24	405.1	46.8	27.74	0.6036007E 05	0.8396718E 02	0.5068264E 07
10	97.4	18.41	487.0	49.6	29.36	0.7629013E 05	0.8043076E 02	0.6136072E 07
11	100.7	20.85	580.9	52.3	30.85	0.9825956E 05	0.7635745E 02	0.7502849E 07
12	105.2	23.00	679.4	55.4	32.60	0.1327806E 06	0.7015376E 02	0.9315056E 07
13	108.7	25.25	777.8	57.8	33.91	0.1681738E 06	0.6602002E 02	0.1110284E 08
14	112.5	27.63	892.0	60.6	35.45	0.2144399E 06	0.6179498E 02	0.1325131E 08
15	116.7	30.51	1023.2	62.9	36.72	0.2787803E 06	0.5751753E 02	0.1603476E 08
16	120.5	32.81	1150.7	65.8	38.30	0.3499571E 06	0.5367110E 02	0.1878258E 08
17	124.5	35.25	1258.5	67.0	38.87	0.4415981E 06	0.4989162E 02	0.2203203E 08
18	128.7	37.12	1420.6	71.8	41.54	0.5585521E 06	0.4584254E 02	0.2560544E 08
19	141.3	38.50	1588.6	77.4	44.25	0.1096891E 07	0.3379160E 02	0.3706570E 08
20	145.6	40.25	1731.8	80.7	45.99	0.1360003E 07	0.3107950E 02	0.4226821E 08
21	151.0	42.41	1888.3	83.5	47.38	0.1744001E 07	0.2818309E 02	0.4915133E 08
22	156.4	44.36	2027.1	85.7	48.43	0.2168718E 07	0.2581285E 02	0.5598078E 08

Table A5.26 Heat Transfer Data -- Soln. 12

Run	T_B °F	ΔT °F	Q Btu/hr	$\bar{h}$ Btu/hrft²°F	$\overline{N_{Nu}}$	N_{Gr}	N_{Pr}'	$N_{Gr}N_{Pr}'$
1	80.3	2.60	15.9	11.5	6.87	0.3555459E 01	0.3790628E 04	0.1347742E 05
2	82.8	4.97	34.6	13.1	7.79	0.7698756E 01	0.3643886E 04	0.2805339E 05
3	86.3	8.35	62.3	14.0	8.33	0.1515254E 02	0.3449927E 04	0.5227518E 05
4	90.5	11.44	99.5	16.3	9.68	0.2562024E 02	0.3188733E 04	0.8169606E 05
5	94.1	14.90	143.9	18.1	10.74	0.3896259E 02	0.3011733E 04	0.1170438E 06
6	98.1	18.70	195.6	19.6	11.60	0.5739191E 02	0.2829993E 04	0.1624161E 06
7	102.7	22.64	254.9	21.1	12.46	0.8510741E 02	0.2613396E 04	0.2224193E 06
8	107.2	26.72	326.6	22.9	13.49	0.1210383E 03	0.2426976E 04	0.2937572E 06
9	111.9	30.60	404.7	24.8	14.55	0.1695927E 03	0.2237424E 04	0.3794507E 06
10	116.3	34.06	485.7	26.8	15.65	0.2293588E 03	0.2066498E 04	0.4739695E 06
11	122.1	37.90	583.5	28.9	16.83	0.3365164E 03	0.1843080E 04	0.6202265E 06
12	126.5	41.21	676.0	30.8	17.88	0.4404055E 03	0.1706449E 04	0.7515298E 06
13	132.6	46.19	785.5	31.9	18.47	0.6306018E 03	0.1540517E 04	0.9714527E 06
14	136.9	49.03	881.4	33.7	19.46	0.8050151E 03	0.1425075E 04	0.1147206E 07
15	148.6	50.54	1017.9	37.8	21.57	0.1585702E 04	0.1078260E 04	0.1709799E 07
16	153.8	55.03	1157.6	39.5	22.47	0.2045370E 04	0.1002937E 04	0.2051377E 07
17	158.4	58.36	1282.7	41.2	23.41	0.2558053E 04	0.9339771E 03	0.2389163E 07
18	163.2	62.42	1423.2	42.8	24.21	0.3188489E 04	0.8740852E 03	0.2787011E 07
19	169.6	65.12	1585.1	45.7	25.73	0.4431367E 04	0.7699756E 03	0.3412044E 07
20	174.0	67.41	1705.9	47.5	26.66	0.5597797E 04	0.7044109E 03	0.3943149E 07

Table A5.27 Heat Transfer Data -- Soln. 13

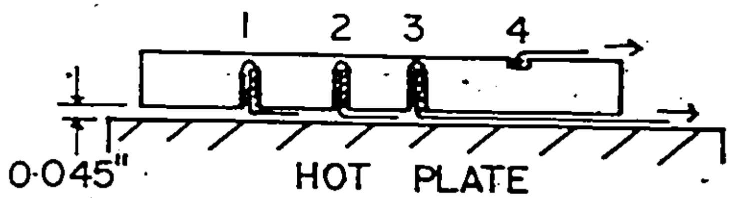
Run	T_s °F	ΔT °F	Q Btu/hr	$\bar{h}$ Btu/hrft^2°F	$\overline{N_{Nu}}$	N_{Gr}	N_{Pr}'	$N_{Gr}N_{Pr}'$
1	76.2	1.49	4.0	5.0	3.01	0.1980977E-01	0.3046843E 05	0.6035723E 03
2	79.2	4.79	15.4	6.0	3.62	0.7806224E-01	0.2959264E 05	0.2310067E 04
3	83.1	8.53	34.8	7.6	4.57	0.1682243E 00	0.2828740E 05	0.4758629E 04
4	88.8	13.93	60.4	8.1	4.84	0.3497347E 00	0.2647119E 05	0.9257891E 04
5	94.0	18.04	101.7	10.6	6.27	0.5683427E 00	0.2466098E 05	0.1401589E 05
6	99.4	22.82	146.0	12.0	7.10	0.8955235E 00	0.2298621E 05	0.2058469E 05
7	105.3	27.35	195.4	13.4	7.90	0.1391133E 01	0.2104646E 05	0.2927844E 05
8	112.7	32.64	250.6	14.4	8.45	0.2323336E 01	0.1868964E 05	0.4342232E 05
9	119.1	38.44	327.5	16.0	9.34	0.3513775E 01	0.1710470E 05	0.6010207E 05
10	126.4	44.49	405.6	17.1	9.95	0.5507600E 01	0.1530776E 05	0.8430900E 05
11	132.3	49.43	488.5	18.5	10.75	0.7860057E 01	0.1394388E 05	0.1095996E 06
12	138.5	54.50	582.0	20.0	11.58	0.1120572E 02	0.1265404E 05	0.1417976E 06
13	145.4	60.45	683.5	21.2	12.21	0.1644095E 02	0.1137506E 05	0.1870168E 06
14	152.9	65.36	784.3	22.5	12.89	0.2525433E 02	0.9920176E 04	0.2505274E 06
15	159.1	69.39	886.3	24.0	13.67	0.3533293E 02	0.8904691E 04	0.3146288E 06
16	167.0	75.18	1021.2	25.5	14.46	0.5232158E 02	0.7885410E 04	0.4125771E 06
17	180.9	80.57	1134.4	26.4	14.83	0.9489261E 02	0.6437461E 04	0.6108674E 06

APPENDIX VI

ERROR ANALYSIS

A. Test for Conduction Effect on Thermocouples

AWG 30 copper-constantan thermocouples were embedded in a 1/4 inch flat copper plate as shown in Figure A6.1.



FIG·A6·1

TEST FOR CONDUCTION EFFECT

Three methods of installing the thermocouples were tested.

Thermocouple 1 was first glued into a screw through a centre hole bored along the vertical axis of the screw, with the thermocouple probe protruding from the lower end of the screw. The thermocouple was then screwed into the threaded hole until the thermocouple probe was in good contact with the copper plate. Thermocouple 2 was glued into the hole with copper cement. Thermocouple 3 was soldered into a brass pin and the pin was forced into the hole. Thermocouple 4 was soldered into the copper plate.

The copper plate sat on a hot plate with the leading wires of thermocuoples 1, 2 and 3 sandwiched between the hot plate and copper plate which was 0.045 inch apart. The following data is obtained and shown in Table A6.1.

Table A6.1 Conduction Effect on Thermocouples

	Thermocouple 1	2	3	4
Room temperature, mv	0.980	0.980	0.980	0.980
Temperature of hot plate, mv	7.385	7.320	7.106	7.081
Error, %	4.3	3.1	0.2	

The deviation of thermocouple 3 from true temperature which was measured by thermocouple 4, was much lower than the other two thermocouples. The error could still be reduced by embedding the thermocouple leads into the copper plate as described in Chapter IV.

B. Heat Loss from the Sphere

The heat loss from the sphere to the surrounding air and fluid would be mainly through the teflon tube, copper and constantan wires both from the power supply line and thermocouples. Assume that all the heat loss is travelling upward through 2.5 feet of the supporting rod into the air above the solution. This is especially true for the copper and constantan wires because they are insulated. The following heat loss calculation is based on the highest heat input to the sphere, which is about 1900 Btu/hr .

(1) Teflon Tube

Cross sectional area Ar of the tube with ID = 3/8 inch and OD = 5/8 inch is 1.363×10^{-2} ft.^{2}

k = 0.2418 Btu/hr. ft.°F.

$$q = kAr \frac{\Delta T}{\Delta x} = 0.13 \text{ Btu/hr.}$$

(2) Copper wire from gage 16 power supply line

Ar = 5.65×10^{-5} ft.^{2}

ΔT = 750°F

k = 210 Btu/hr. ft°F

q was found to be 3.6 Btu/hr.

(3) Thermocouple wires

Assume the thermal conductivity of constantan is equal to that of copper .

(i) Five AWG 36 thermocouple

Total Ar = 1.36×10^{-6} ft.^{2}

ΔT = 100°F

q was found to be 0.01 Btu/hr.

(ii) AWG 24 thermocouple

Ar = 4.41×10^{-5} ft^2

ΔT = 100°F

q was found to be 0.04 Btu/hr.

Therefore a total heat of 3.78 Btu/hr., which was about 0.2% of the heat input to the sphere, was considered negligible and no corrections were made in treating the experimental results.

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1962	Completed high school at Ipoh, Perak, Malaysia.
1967	Received the Bachelor of Applied Science in Chemical Engineering from National Taiwan University, Taiwan, Republic of China.
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1973	Presently a candidate for the degree of Ph.D. in Chemical Engineering.